18th Autumn Workshop on Number Theory

“Geometrical Applications of Modular Forms of Several Variables”

Atsuhira Nagano (Waseda Univ.)
”Modular functions coming from elliptic curves and K3 surfaces”

Abstract: Elliptic modular functions are very important in mathematics. In old times, elliptic modular functions appeared in the study of elliptic curves. They were studied by Gauss, Jacobi, Kronecker, etc. At that time, modular functions were closely related to elliptic integrals, hypergeometric functions and class fields of imaginary quadratic fields. In this talk, we will see the above classical results. Moreover, K3 surfaces give 2-dimensional analogy of elliptic curves. We can obtain non-trivial modular functions coming from K3 surfaces. In this talk, we will see basic properties of K3 surfaces.

Atsuhira Nagano (Waseda Univ.)
”Hilbert modular functions via K3 surfaces and applications in algebraic number theory”

Abstract: Hilbert modular functions were first studied by Blumenthal and Hecke. Since then, they have been important in algebraic geometry and number theory. The speaker will present a result of the Hilbert modular functions for the minimal discriminant via K3 surfaces. This result has applications in number theory. Namely, the period mappings of K3 surfaces allow us to obtain new explicit models of Shimura curves and a simple construction of class fields over quartic CM-fields.

Shouhei Ma (Tokyo Institute of Technology)
"Birational type of orthogonal modular varieties"

Abstract: My talk will be about the birational type of modular varieties defined by orthogonal groups of signature $(2, n)$. I will talk about results to the effect that these modular varieties are often of general type in higher dimension, which is analogous to the Tai-Freitag-Mumford theorem in the symplectic case. A feature of the orthogonal case is the appearance of the branch divisor. For the proof I will follow the approach proposed by Gritsenko-Hulek-Sankaran (2008), which is a combination of the Jacobi lifting and estimate of the (Hirzebruch-Mumford) volume of orthogonal groups.

Shigeyuki Kondo (Nagoya Univ.)
"An application of Borcherds products"

Abstract: Borcherds gave two liftings called additive and multiplicative one which are automorphic forms on a bounded symmetric domain of type IV. In this talk, I will give an application of his theory to Algebraic Geometry. I have the following examples: moduli of plane quartics (=non-hyperelliptic curves of genus 3), moduli of ordered 8 points on the projective line (hyper elliptic curves of genus 3), ordered 6 points on the projective line (= curves of genus 2), moduli of cubic surfaces (Allcock-Freitag), moduli of Enriques surfaces. I will discuss one example. For example, in case of 8 points, this moduli space is an arithmetic quotient of a 5-dimensional complex ball. It follows from Borcherds theory that there exists a 14-dimensional space of automorphic forms on the complex ball. This space gives an embedding of the moduli space into 13-dimensional projective space. This embedding coincides with the one given by cross ratios of 8 points on the projective line.

Ken-Ichi Yoshikawa (Kyoto Univ.)
"Analytic torsion for K3 surfaces with involution"

Abstract: A holomorphic torsion invariant of K3 surfaces with involution was introduced by the speaker nearly 20 years ago. Its automorphy on the moduli space and an explicit formula on the components of the moduli space of dimension¡10 have been known so far. Very recently, together with S. Ma, we could determine the structure of the invariant for all components of the moduli space. In my talk, I will explain the construction of the holomorphic torsion invariant and its explicit formula as automorphic function on the moduli space. On each component of the moduli space, our invariant is always expressed by the product of an explicit Borcherds product and a classical Siegel modular form. If time allows, I will also explain the physics conjecture behind our invariant, i.e., the mirror symmetry at genus 1.

Hironori Shiga (Chiba Univ.)
"Period map, Picard modular forms and Complex multiplication of higher degree" (Joint work with Atsuhira Nagano (Waseda Univ.))

Abstract: In this talk we aim to show one explicit example of the theory of complex multiplication of higher degree by Goro Shimura that gives the Hilbert class fields of some CM fields. The theory is big and precise so that it is beyond the speaker's poor capability. But, up to now, there has not been presented any explicit example of the Hilbert class field via this theory. Our result is concerned with the theory of Picard modular forms and their restrictions to one dimensional loci. So we shall give precise explanation of the connection among the period map, Fuchsian differential equations and modular forms for arithmetic triangle groups. Under this preparation we can state the results in an explicit way. Still if we would have some more minutes, we wish to speak about the relation be tween Picard modular forms and K3 modular forms.

Kyoji Saito (MPIM)
"Automorphic forms associated with a period map of primitive forms"

Abstract: No abstract.