Odd perfect numbers have a prime factor exceeding 10^8 *

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Abstract

In this article, it is shown that every odd perfect number is divisible by a prime greater than 10^8. The method of the proof is based on the paper by Hagis and Cohen. By improving on the original algorithms, we could shorten computing time. Some property of cyclotomic polynomials is used in our algorithms.

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1 Introduction

A positive integer n is said to be *perfect* if $\sigma(n) = 2n$, where $\sigma(n)$ denotes the sum of positive divisors of n. As of January 2006, forty three even perfect numbers are known, however, it is still open whether or not odd one does exist. Many necessary conditions for their existence have been found. For example, Euler showed that the prime decomposition of an odd perfect number n must be the form of

$$n = p^e p_1^{2e_1} \cdots p_t^{2e_t}, \quad p \equiv e \equiv 1 \pmod 4.$$

This p is called the *special prime* of n. Brent, Cohen and te Riele [1] showed that $n > 10^{300}$. Hagis [6] and Chein [2] independently showed that n must have at least 8 distinct prime factors, and Hare [8] showed that n must have totally at least 47 prime factors.

In this article, we focus our attention on the largest prime factor of an odd perfect number. Suppose that n is an odd perfect number and P_n is the largest prime factor of n. In 1944, Kanold [11] showed that $P_n > 60$[§]. This bound was extended by Hagis and McDaniel [4] (resp. [5]) to $P_n > 10^4$ (resp. $P_n > 10^5$), by Hagis and Cohen [7] to $P_n > 10^6$, by Jenkins [9], [10] to $P_n > 10^7$. Jenkins reported that he needed about 25800 hours for computing time. The aim of this article is to show the following result.

Theorem 1.1 *If n is an odd perfect number and P_n is the largest prime factor, then $P_n > 10^8$.*

In order to show this theorem, it was necessary to improve on the original algorithms. In §3.1, we discuss the improvement of the algorithm, and give the data of CPU time in §A.2.

2 Cyclotomic numbers

Let d be a positive integer. The d-th cyclotomic polynomial $\Phi_d(X)$ is defined by

$$\Phi_d(X) = \prod_{\substack{j \in \{1, \cdots, d\} \\ (j, d) = 1}} (X - e^{2\pi\sqrt{-1}j/d}).$$

It is well-known that $\Phi_d(X) \in \mathbb{Z}[X]$. We often deal with the case that d is equal to a prime r, in which

$$\Phi_r(X) = X^{r-1} + X^{r-2} + \cdots + X + 1.$$

For a positive integer a, the integer $\Phi_d(a)$ is called a *cyclotomic number*. For a prime p, it is easily verified that

$$\sigma(p^e) = \prod_{\substack{d \mid (e+1) \\ d \neq 1}} \Phi_d(p),$$

hence cyclotomic numbers are important for a study of odd perfect numbers. Assume that n is an odd perfect number, and its prime decomposition is $p_1^{e_1} \cdots p_k^{e_k}$. From $2n = \sigma(n)$, it immediately follows that

$$2\prod_{i=1}^{k} p_i^{e_i} = \prod_{i=1}^{k} \prod_{\substack{d \mid (e_i+1) \\ d \neq 1}} \Phi_d(p_i). \tag{2.1}$$

In this section, we recall the properties of cyclotomic numbers.

In 19-th century, cyclotomic numbers were studied by Sylvester, Kronecker et al. The following proposition is a summary of their results (cf. [14], [15]).

[§]It is considered that he did not use a computer. The second author showed that the largest prime factor of an odd *harmonic number* must be greater than 100 (cf. [3]), without a computer. This is an extension of Kanold's result.

Proposition 2.1 *Let p be a prime, and a, d integers. Suppose that $a \geq 2$, $d \geq 3$. Then we have the following.*
(1) $p \mid \Phi_d(a) \Rightarrow p \mid d$ *or* $p \equiv 1 \pmod{d}$.
(2) $p \mid \Phi_d(a)$, $p \mid d \Rightarrow p^2 \nmid \Phi_d(a)$.
(3) *Every cyclotomic number $\Phi_d(a)$ has at least one prime factor p such that $p \equiv 1 \pmod{d}$, except the case that $\Phi_6(2) = 3$.*

Our improved algorithms are based on the following proposition.

Proposition 2.2 *Let p, q, r be primes, and suppose that $q \mid \Phi_r(p)$, $q \equiv 1 \pmod{r}$. Let g be a generator of the cyclic group $(\mathbb{Z}/q^m\mathbb{Z})^\times$, and put $t = (q-1)/r$. Then $q^m \mid \Phi_r(p)$ if and only if p belongs to the subgroup $\langle g^{tq^{m-1}} \rangle$ of $(\mathbb{Z}/q^m\mathbb{Z})^\times$.*

proof. Note that t is an integer since $q \equiv 1 \pmod{r}$. Assume that $p \equiv 1 \pmod{q}$. Then it follows that $\Phi_r(p) \equiv r \pmod{q}$, a contradiction. Hence $p \not\equiv 1 \pmod{q}$ and

$$\begin{aligned} q^m \mid \Phi_r(p) &\iff q^m \mid (p-1)\Phi_r(p) \\ &\iff p^r \equiv 1 \pmod{q^m} \\ &\iff \text{the order of } p \in (\mathbb{Z}/q^m\mathbb{Z})^\times \text{ is } r \\ &\iff p \in \langle g^{tq^{m-1}} \rangle, \end{aligned}$$

as required. □

3 Outline of the proof

In this section, we give an outline of the proof of Theorem 1.1. We discuss details of the proof in the appendix. In this article, p, q, r will always denote primes.

3.1 Acceptable values

Assume that n is an odd perfect number and the largest prime factor of n is less then 10^8. Then the left-hand side of (2.1) has only prime factors less then 10^8, hence so does the right-hand side. Since $2n \equiv 2 \pmod{4}$, the cyclotomic numbers in (2.1) are not divided by 4.

Definition For an odd prime p and a prime r, we say that $\Phi_r(p)$ is *acceptable* if and only if the following two conditions hold.

(1) $\Phi_r(p)$ has no prime factor greater than 10^8.
(2) $4 \nmid \Phi_r(p)$.

Clearly, the cyclotomic numbers in (2.1) must be acceptable. Our first aim (but the hardest task) is to find all acceptable values.

Lemma 3.1 *Suppose that $r \geq 7$, $3 \leq p < 10^8$ and the cyclotomic number $\Phi_r(p)$ is acceptable. Then $\Phi_r(p)$ is one of 671 numbers listed in §A.5.*

The complete proof of this lemma is in §A.1. From Proposition 2.1 (3), the cyclotomic number $\Phi_r(p)$ has a prime factor q such that $q \geq 2r + 1$, hence if $\Phi_r(p)$ is acceptable, then $r < 5 \cdot 10^7$. Therefore we need to investigate only finitely many numbers, however, it is hard to directly judge each number acceptable (or unacceptable) because of difficulty of prime factorization. The key point of the proof of Lemma 3.1 is the following fact. In this article, $p^e \parallel n$ means that $p^e \mid n$ and $p^{e+1} \nmid n$.

Lemma 3.2 *If $p, q < 10^8$, $6679 < r < 5 \cdot 10^7$, then $q^4 \nmid \Phi_r(p)$, and $q^3 \parallel \Phi_r(p)$ for at most one q for each $\Phi_r(p)$. In fact, $q^3 \nmid \Phi_r(p)$ except that*

$$28499^3 \parallel \Phi_{14249}(70081199), \quad 60647^3 \parallel \Phi_{30323}(6392117), \quad 63587^3 \parallel \Phi_{31793}(42326917).$$

We can check this fact using UBASIC program `cubeprg.ub` or PARI/GP program `cubeprg.gp`. Since checking this fact needs much time, it is important to improve the algorithm. For each pair (r, q) with $q \equiv 1 \pmod{r}$, we need to search p such that $q^2 \mid \Phi_r(p)$. The original algorithm moves p for whole range, however, Proposition 2.2 says that $q^2 \mid \Phi_r(p)$ is equivalent to $p \in \langle g^{q(q-1)/r} \rangle \subset (\mathbb{Z}/q^2\mathbb{Z})^\times$, hence, we need move p for only this range. This improvement shortens CPU time very much. For details, see §A.2.

3.2 Inadmissible small primes

Under Lemma 3.1, we show the following.

Lemma 3.3 *Suppose that an odd perfect number n has no prime factor greater than 10^8. Then n has no prime factor which is in the following set X.*

$$X = \{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 43, 61, 71, 113, 127, 131, 151, 197, 211, 239, 281, 337, \\ 379, 421, 449, 463, 491, 547, 617, 631, 659, 673, 743, 757, 827, 911, 953, 967, 1051, 1093\}.$$

In particular, n has no prime factor less than 41.

For the proof of this lemma, primes in X are considered in the order

$$1093, 151, 31, 127, 19, 11, 7, 23, 131, 37, 61, 13, 3, 5, 29, 43, 1051, 17, 71, 113, 197, 211, 239 \\ 281, 337, 379, 421, 449, 463, 491, 547, 617, 631, 659, 673, 743, 757, 827, 911, 953, 967.$$

For example, after proving that $1093 \nmid n$, we prove $151 \nmid n$ as follows. Assume that $151 \mid n$. Then the right-hand side of (2.1) is divided by a cyclotomic number $\Phi_r(151)$. Only $\Phi_3(151) = 3 \cdot 7 \cdot 1093$ is acceptable, however, it is contradictory to $1093 \nmid n$. Hence we have $151 \nmid n$. The complete proof is given in §A.3.

3.3 Restriction on exponents in the prime decomposition

Under Lemmas 3.1, 3.3, we show the following.

Lemma 3.4 *Suppose that an odd perfect number n has no prime factor greater than 10^8, and the prime decomposition of n is $n = p_1^{e_1} \cdots p_r^{e_r}$. Then the product $\prod(e_i + 1)$ has no prime factor greater than 5.*

Assume that some $e_i + 1$ has a prime factor r greater than 5. Then the cyclotomic number $\Phi_r(p_i)$ is acceptable. Hence $\Phi_r(p_i)$ is one of the numbers listed in §A.5 by Lemma 3.1. From Lemma 3.3, it follows that $p_i \notin X$, and $\Phi_r(p_i)$ has no prime factor which is in X. There are 87 such cyclotomic numbers. It is sufficient to show that such numbers do not appear in the equation (2.1).

In this section, we check only three $\Phi_r(p_i)$ with $r \geq 11$, and check other numbers in §A.4.

$$\Phi_{13}(47) = 53 \cdot 2237 \cdot 14050609 \cdot 71265169,$$
$$\Phi_{13}(83) = 1249 \cdot 1396513 \cdot 1423319 \cdot 43580447,$$
$$\Phi_{11}(691) = 59951 \cdot 133717 \cdot 183041 \cdot 455489 \cdot 37187767.$$

(1) If $\Phi_{13}(47) \mid n$, then $14050609 \mid n$. Only $\Phi_2(14050609) = 2 \cdot 5 \cdot 7 \cdot 200723$ is acceptable, however, $5 \in X$, a contradiction.

(2) If $\Phi_{13}(83) \mid n$, then $1396513 \mid n$. Only $\Phi_2(1396513) = 2 \cdot 7 \cdot 23 \cdot 4337$ is acceptable, however, $7 \in X$, a contradiction.
(3) If $\Phi_{11}(691) \mid n$, then $133717 \mid n$. Only $\Phi_2(133717) = 2 \cdot 13 \cdot 37 \cdot 139$ is acceptable, however, $13 \in X$, a contradiction.

3.4 Four sets

In this section, we complete the proof of Theorem 1.1. Let $P = \{p \,|\, 41 \leq p < 10^8\}$. Using the program `ptester.ub`, we have $\#P = 5761443$ and

$$P^* := \prod_{p \in P} \frac{p}{p-1} < 4.87934286481804236682.$$

We define subsets S, T, U, V of P as follows.

$$\begin{aligned}
S &= \{p \in P \mid p \not\equiv 1 \pmod 3 \text{ and } p \not\equiv 1 \pmod 5\}, \\
T &= \{p \in P \mid p \equiv 1 \pmod{15}\}, \\
U &= \{p \in P \mid p \equiv 1 \pmod 3,\ p \not\equiv 1 \pmod 5 \text{ and } \Phi_5(p) \text{ has a prime factor greater than } 10^8\}, \\
V &= \{p \in P \mid p \not\equiv 1 \pmod 3,\ p \equiv 1 \pmod 5 \text{ and } \Phi_3(p) \text{ has a prime factor greater than } 10^8\}.
\end{aligned}$$

Note that these subsets are disjoint. Using the programs `stester.ub`, `ttester.ub`, `utester.ub` and `vtester.ub`, we have $\#S = 2160618$, $\#T = 719983$, $\#U = 2144188$, $\#V = 496701$ and

$$\begin{aligned}
S^* &:= \prod_{p \in S} \frac{p}{p-1} > 1.82219345901032950583, \\
T^* &:= \prod_{p \in T} \frac{p}{p-1} > 1.19902263543776496408, \\
U^* &:= \prod_{p \in U} \frac{p}{p-1} > 1.43699138263382743310, \\
V^* &:= \prod_{p \in V} \frac{p}{p-1} > 1.03750936160818766647.
\end{aligned}$$

The following proposition can be proven similarly to Hagis and Cohen [7].

Proposition 3.5 *Suppose that an odd perfect number n has no prime factor greater than 10^8. Then the following hold.*
(1) *The number n is divisible by at most two elements of S. If there is such an element s, then it is not the special prime of n, and $s \geq 47$.*
(2) *The number n is divisible by at most one element of T. If there is such an element t, then it is the special prime of n, and $t \geq 61$.*
(3) *The number n is divisible by at most one element of U. If there is such an element u, then it is the special prime of n, and $u \geq 73$.*
(4) *The number n is not divisible by any element of V.*

proof. We show only (1) since it is slightly different from the original one. Suppose that $p \in S$ and $p \mid n$. Then p divides a cyclotomic number $\Phi_d(p_j)$ in the equality (2.1). Assume that p_j is not the special prime. Then d is odd. From Lemma 3.4, d is divisible by only 3 or 5, hence, $p \equiv 1 \pmod 3$ or $p \equiv 1 \pmod 5$ by Proposition 2.1. This is a contradiction to $p \in S$. Therefore p_j is the special prime, and p is not special. Since the smallest element of S is 47, it holds that $p \geq 47$. Since p is not special and divides only $\Phi_2(p_j)$, we have $p^2 \mid (p_j + 1)$. If there are three such p, then $p_j + 1 \geq 2 \cdot 47^2 \cdot 53^2 \cdot 59^2 > 10^8$, a contradiction to $p_j < 10^8$. □

We define $\sigma_{-1}(n)$ by

$$\sigma_{-1}(n) := \sum_{d|n} d^{-1} = \frac{\sigma(n)}{n}.$$

The integer n is perfect if and only if $\sigma_{-1}(n) = 2$. The function σ_{-1} is multiplicative, that is, if $(a, b) = 1$, then $\sigma_{-1}(ab) = \sigma_{-1}(a)\sigma_{-1}(b)$. It is easily verified that if $p < q$, then $\sigma_{-1}(p^e) > \sigma_{-1}(q^f)$ for any positive integers e, f, and

$$\sigma_{-1}(p^e) < \sigma_{-1}(p^\infty) := \lim_{m\to\infty} \sigma_{-1}(p^m) = \frac{p}{p-1}.$$

Assume that n is an odd perfect number whose largest prime factor is less than 10^8. From Proposition 3.5, it follows that

$$2 = \sigma_{-1}(n) < \prod_i \frac{p_i}{p_i - 1} \leq \frac{47}{46} \cdot \frac{53}{52} \cdot \frac{61}{60} \cdot \frac{P^*}{S^*T^*U^*V^*} < 1.5859314817,$$

a contradiction. The proof of Theorem 1.1 is completed.

This inequality is much stronger than is needed and that the theorem could be proved by only S and U. The referee of the paper [7] also pointed out the same fact. However, it is considered that the four sets are valuable, as Hagis and Cohen claimed.

A Appendix

A.1 Details on the search for acceptable values

In the rest of this article, p, q, r will always denote primes less than 10^8.

Lemma A.1 *If $10^2 < p < 10^8$ and $6679 < r < 5 \cdot 10^7$, then $\Phi_r(p)$ is unacceptable.*

proof. For a prime r, we define $Q(r)$ by

$$Q(r) = \prod_{p<10^8,\, p\equiv 1 \pmod r} p.$$

First, we show

Claim 1. If $6679 < r < 5 \cdot 10^8$, then $(10^8)^2 \cdot r \cdot Q(r)^2 < 10^{2(r-1)}$.

Suppose that $r \geq 5 \cdot 10^4$. If $p \equiv 1 \pmod r$, then $p = 2kr + 1$ with $k < 10^3$, hence we have $10^8 \cdot \sqrt{r} \cdot Q(r) < (10^8)^{10^3+2} < 10^{10^4} < 10^{r-1}$, as required. The case that $6679 < r < 5 \cdot 10^4$ can be directly checked by the program `claim1.ub`.

Assume that $6679 < r < 5 \cdot 10^7$ and $\Phi_r(p)$ is acceptable. From Lemma 3.2, it follows that

$$\Phi_r(p) < (10^8)^2 \cdot r \cdot Q(r)^2 < 10^{2(r-1)}.$$

On the other hand, we have

$$\Phi_r(p) > p^{r-1} > 10^{2(r-1)},$$

a contradiction. □

Lemma A.2 *If $10^2 < p < 10^8$ and $4723 < r \leq 6679$, then $\Phi_r(p)$ is unacceptable.*

proof. By using the program `claim2.ub`, we can show

Claim 2. If $4723 < r \leq 6679$, then $10^8 r \cdot Q(r) < 10^{2(r-1)}$.

By the program `sqrprg.ub`, we can show

If $4723 < r \le 6679$, then $q^3 \nmid \Phi_r(p)$, and $q^2 \parallel \Phi_r(p)$ for at most one q for each $\Phi_r(p)$.

Assume that $4723 < r \le 6679$ and $\Phi_r(p)$ is acceptable. Then it follows that $10^{2(r-1)} < p^{r-1} < \Phi_r(p) < 10^8 \cdot r \cdot Q(r) < 10^{2(r-1)}$, a contradiction. □

Lemma A.3 (1) *If* $10^6 < p < 10^8$, $2707 < r \le 4723$, *then* $\Phi_r(p)$ *is unacceptable.*
(2) *If* $10^7 < p < 10^8$, $2503 < r \le 2707$, *then* $\Phi_r(p)$ *is unacceptable.*

proof. By the program `sqrprg.ub`, we can show that if $3 \le p, q < 10^8$, $2503 < r \le 4723$, then $q^2 \mid \Phi_r(p)$ for at most one q for each $\Phi_r(p)$, and $q^3 \nmid \Phi_r(p)$ expect that

$$10709^3 \parallel \Phi_{2677}(6619441), \quad 5939^3 \parallel \Phi_{2969}(41492783), \quad 6719^3 \parallel \Phi_{3359}(59698039),$$
$$8147^3 \parallel \Phi_{4073}(41112823), \quad 8147^3 \parallel \Phi_{4073}(41728717).$$

(1) By the program `claim3.ub`,

Claim 3. If $2707 < r \le 4723$, then $(10^8)^2 \cdot r \cdot Q(r) < 10^{6(r-1)}$.

Hence if $2707 < r \le 4723$ and $\Phi_r(p)$, then it follows that $10^{6(r-1)} < p^{r-1} < \Phi_r(p) < (10^8)^2 \cdot r \cdot Q(r) < 10^{6(r-1)}$, a contradiction.
(2) By the program `claim4.ub`,

Claim 4. If $2503 < r \le 2707$, then $(10^8)^2 \cdot r \cdot Q(r) < 10^{7(r-1)}$.

Hence if $2503 < r \le 2707$ and $\Phi_r(p)$, then it follows that $10^{7(r-1)} < p^{r-1} < \Phi_r(p) < (10^8)^2 \cdot r \cdot Q(r) < 10^{7(r-1)}$, a contradiction. □

Suppose that $p < 10^2$. If $q^2 \mid \Phi_r(p)$, then $r \mid (q-1)$ and $p^r \equiv 1 \pmod{q^2}$, hence $p^{q-1} \equiv 1 \pmod{q^2}$. Conversely if $p^{q-1} \equiv 1 \pmod{q^2}$ and there is a prime factor r of $q-1$ such that $p^r \equiv 1 \pmod{q^2}$, then $q^2 \mid \Phi_r(p)$. According to the table of Montgomery [12], all solutions of $p^{q-1} \equiv 1 \pmod{q^2}$, $3 \le p < 10^2$, $q < 10^8$ are given in the following table.

p	q	p	q
3	11, 1006003	43	5, 103
5	20771, 40487, 53471161	47	
7	5, 491531	53	3, 47, 59, 97
11	71	59	2777
13	863, 1747591	61	
17	3, 46021, 48947	67	7, 47, 268573
19	3, 7, 13, 43, 137, 63061489	71	3, 47, 331
23	13, 2481757, 13703077	73	3
29		79	7, 263, 3037, 1012573, 60312841
31	7, 79, 6451, 2806861	83	4871, 13691
37	3, 77867	89	3, 13
41	29, 1025273	97	7, 2914393

From the table, if $p < 10^2$, then $q^2 \nmid \Phi_r(p)$ except that

$$11^2 \parallel \Phi_5(3),\ 48947^2 \parallel \Phi_{24473}(17),\ 47^2 \parallel \Phi_{23}(53),\ 59^2 \parallel \Phi_{29}(53),$$
$$7^2 \parallel \Phi_3(67),\ 47^2 \parallel \Phi_{23}(71),\ 7^2 \parallel \Phi_3(79),\ 4871^2 \parallel \Phi_{487}(83).$$

¶We also checked this by the program `cubeprg.gp`. The computer search showed that if $6679 < r < 5 \cdot 10^7$, $q < 10^8$, $a < 10^2$, then $q^2 \nmid \Phi_r(a)$ except that $48947^2 \parallel \Phi_{24473}(17)$, $401771^2 \parallel \Phi_{40177}(63)$.

For a prime p, we define $R(p)$ by

$$R(p) = \min\{a \in \mathbb{N} \mid r \geq a \Rightarrow 10^8 r \cdot Q(r) < p^{r-1}\}.$$

It immediately follows that $R(p) \leq 5 \cdot 10^4$. In fact, if $r \geq 5 \cdot 10^4$, then

$$10^8 r \cdot Q(r) < (10^8)^{10^3+2} < (3^{17})^{10^3+2} < 3^r \leq p^r.$$

By directly checking each $r < 5 \cdot 10^4$ (`rvalue.ub`), we have the following table.

p	$R(p)$	p	$R(p)$	p	$R(p)$
3	9650	29	5508	61	4952
5	7950	31	5508	67	4890
7	7238	37	5310	71	4878
11	6548	41	5262	73	4878
13	6318	43	5262	79	4818
17	5954	47	5108	83	4788
19	5882	53	5060	89	4734
23	5660	59	4988	97	4724

Lemma A.4 *If $3 \leq p < 10^2$ and $R(p) \leq r < 5 \cdot 10^7$, then $\Phi_r(p)$ is unacceptable.*

proof. Assume that $\Phi_r(p)$ is acceptable. Since $q^2 \nmid \Phi_r(p)$ for at most one q, it follows that $p^r < \Phi_r(p) < 10^8 r \cdot Q(r) < p^r$, a contradiction. □

Now, we must check only r in the table below for each p.

p	r
$3 \cdots 10^2$	$R(p) - 1$
$10^5 \cdots 10^6$	4723
$10^6 \cdots 10^7$	2707
$10^7 \cdots 10^8$	2503

Using `accept.ub` for $7 \leq r \leq 4723$, `accept2.ub` for $3 \leq p < 10^2$, we can show Lemma 3.1.

A.2 Remarks on CPU time

We used the following two systems for the computations.

	machine	CPU	OS	software
A:	HP, AlpherServer GS320	Alpha21264, 731MHz	Tru64 UNIX V5.1	PARI/GP, GP2C
B:	DELL, Dimension8300	Pentium4, 3GHz	Windows XP Home Edition	UBASIC

The machine of A belongs to Computing and Communications Center, Kyushu University, one of B to the first author. We can use some CPU's simultaneously in the system A, and "CPU time" means the total time. We used the system A for checking Lemma 3.2, and the system B for the other computations.

Note that GP2C is available on only UNIX, and UBASIC on only Windows. The GP script `cubeprg.gp` in §A.6.1 is available on both UNIX and Windows. For the practical computation on system A, we used `cubeprg1.gp.c` which is a translated C program from `cubeprg1.gp` by GP2C.

In order to observe how faster our algorithm is than the original one, we compared CPU time in the following three conditions.

I. system A and the C program `cubeprg1.gp.c`

II. system B and the UBASIC program `cubeprg.ub`

III. system B and the original program[‖]

Using each condition, we showd the following lemmas which are necessary for each bound $10^5, 10^6, 10^7, 10^8$.

Lemma A.5 *If* $p, q < 10^5$ *and* $211 < r < 5 \cdot 10^4$*, then* $q^3 \nmid \Phi_r(p)$*.*

Lemma A.6 (cf. [7]) *If* $p, q < 10^6$ *and* $659 < r < 5 \cdot 10^5$*, then* $q^3 \nmid \Phi_r(p)$ *except that*

$$3119^3 \parallel \Phi_{1559}(146917).$$

Lemma A.7 ([9],[10]) *If* $p, q < 10^7$ *and* $2142 < r < 5 \cdot 10^6$*, then* $q^3 \nmid \Phi_r(p)$ *except that*

$$10709^3 \parallel \Phi_{2677}(6619441), \quad 60647^3 \parallel \Phi_{30323}(6392117).$$

Lemma A.8 (Lemma 3.2) *If* $p, q < 10^8$ *and* $6679 < r < 5 \cdot 10^7$*, then* $q^3 \nmid \Phi_r(p)$ *except that*

$$28499^3 \parallel \Phi_{14249}(70081199), \quad 60647^3 \parallel \Phi_{30323}(6392117), \quad 63587^3 \parallel \Phi_{31793}(42326917).$$

The CPU time is as follows. The dashes (—) mean that the authors cannot test because of their hard tasks.

	I	II	III
Lemma A.5	about 3 minutes	about 30 seconds	about 5 minutes
Lemma A.6	about 3 hours	about 35 minutes	about 11 hours
Lemma A.7	about 274 hours	about 42 hours	—
Lemma A.8	about 26000 hours[**]	—	—

The list below gives CPU time of the other computations.

	program	CPU time
1.	`sqrprg.ub`	about an hour
2.	`rvalue.ub`	about 30 minutes
3.	`claim1.ub`	about a minute
4.	`claim2.ub`	less than a minute
5.	`claim3.ub`	less than a minute
6.	`claim4.ub`	less than a minute
7.	`accept.ub`	about 570 hours
8.	`accept2.ub`	about 200 minutes
9.	`ptester.ub`	about 30 minutes
10.	`stester.ub`	about 10 minutes
11.	`ttester.ub`	about 5 minutes
12.	`utester.ub`	about 2 hours
13.	`vtester.ub`	about an hour

Proposition 2.2 is also used in program 1,7,12 and 13, and shortens CPU time very much.

In order to renew the record to 10^9, we need much CPU time or a better method. It is shown that if $\Phi_r(p)$ is acceptable, then $r \le 47$. Can we eliminate the possibilities of $47 < r < 5 \cdot 10^7$ without hard computations? The authors consider that a hint is in the paper by Murty and Wong [13]. They showed that if ABC conjecture is true, then largest prime factors of cyclotomic numbers are large enough in a sense.

[‖] Jenkins' program can be downloaded in his web page. See [10].

[**] It takes about four months for this computation since we used ten CPU's simultaneously.

A.3 Proof of Lemma 3.3

The method of writing proof is according to Hagis and Cohen [7] or Jenkins [9]. Each line begins by listing of the primes that are assumed to be factors of n, and is continued by listing of acceptable values. If there is no acceptable value, then we say "*inadmissible*". We write p^* to indicate that p is the special prime. Note that two different primes cannot be special simultaneously.

A. $1093 \nmid n$.

A, $1093:\ \Phi_2(1093) = 2 \cdot 547;\ \Phi_3(1093) = 3 \cdot 398581$.

A, $1093^*, 547:\ \Phi_3(547) = 3 \cdot 163 \cdot 613$.

A, $1093^*, 547, 613:\ \Phi_3(613) = 3 \cdot 7 \cdot 17923;\ \Phi_5(613) = 131 \cdot 20161 \cdot 53551$.

A, $1093^*, 547, 613, 17923:\ \Phi_3(17923) = 3 \cdot 13 \cdot 31 \cdot 265717$.

A, $1093^*, 547, 613, 17923, 265717:$ 265717 is inadmissible.

A, $1093^*, 547, 613, 20161:$ 20161 is inadmissible.

A, $1093, 398581:\ \Phi_2(398581) = 2 \cdot 17 \cdot 19 \cdot 617;\ \Phi_3(398581) = 3 \cdot 1621 \cdot 32668561$.

A, $1093, 398581^*, 617:\ \Phi_3(617) = 97 \cdot 3931$.

A, $1093, 398581^*, 617, 3931:\ \Phi_3(3931) = 3 \cdot 7 \cdot 31 \cdot 23743$.

A, $1093, 398581^*, 617, 3931, 23743:\ \Phi_3(23743) = 3 \cdot 37 \cdot 5078863$.

A, $1093, 398581^*, 617, 3931, 23743, 5078863:$ 5078863 is inadmissible.

A, $1093, 398581, 32668561:\ \Phi_2(32668561) = 2 \cdot 19 \cdot 43 \cdot 19993$.

A, $1093, 398581, 32668561^*, 19993:\ \Phi_3(19993) = 3 \cdot 73 \cdot 1825297$.

A, $1093, 398581, 32668561^*, 19993, 1825297:\ \Phi_3(1825297) = 3 \cdot 326863 \cdot 3397663$.

A, $1093, 398581, 32668561^*, 19993, 1825297, 326863:\ \Phi_3(326863) = 3 \cdot 67 \cdot 3313 \cdot 160441$.

A, $1093, 398581, 32668561^*, 19993, 1825297, 326863, 3313:\ \Phi_3(3313) = 3 \cdot 7 \cdot 7 \cdot 19 \cdot 3931$.

A, $1093, 398581, 32668561^*, 19993, 1825297, 326863, 3313, 3931:\ \Phi_3(3931) = 3 \cdot 7 \cdot 31 \cdot 23743$.

A, $1093, 398581, 32668561^*, 19993, 1825297, 326863, 3313, 3931, 23743:\ \Phi_3(23743) = 3 \cdot 37 \cdot 5078863$.

A, $1093, 398581, 32668561^*, 19993, 1825297, 326863, 3313, 3931, 23743, 5078863:$ 5078863 is inadmissible.

B. $151 \nmid n$.

B, $151:\ \Phi_3(151) = 3 \cdot 7 \cdot 1093$.

B, $151, 1093:$ contradiction to A.

C. $31 \nmid n$.

C, $31:\ \Phi_3(31) = 3 \cdot 331;\ \Phi_5(31) = 5 \cdot 11 \cdot 17351;\ \Phi_{13}(31) = 42407 \cdot 2426789 \cdot 7908811$.

C, $31, 331:\ \Phi_3(331) = 3 \cdot 7 \cdot 5233;\ \Phi_5(331) = 5 \cdot 37861 \cdot 63601$.

C, $31, 331, 5233:\ \Phi_2(5233) = 2 \cdot 2617;\ \Phi_3(5233) = 3 \cdot 7 \cdot 31 \cdot 42073,\ \Phi_5(5233) = 2351 \cdot 7741 \cdot 41213191$.

C, $31, 331, 5233^*, 2617:\ \Phi_3(2617) = 3 \cdot 193 \cdot 11833$.

C, $31, 331, 5233^*, 2617, 11833:\ \Phi_3(11833) = 3 \cdot 13 \cdot 199 \cdot 18043$.

C, $31, 331, 5233^*, 2617, 11833, 18043:\ \Phi_3(18043) = 3 \cdot 7 \cdot 15503233$.

C, $31, 331, 5233^*, 2617, 11833, 18043, 15503233:$ 15503233 is inadmissible.

C, $31, 331, 5233, 42073:\ \Phi_2(42073) = 2 \cdot 109 \cdot 193;\ \Phi_3(42073) = 3 \cdot 19 \cdot 409 \cdot 75931$.

C, $31, 331, 5233, 42073^*, 193:\ \Phi_3(193) = 3 \cdot 7 \cdot 1783$.

C, $31, 331, 5233, 42073^*, 193, 1783:\ \Phi_3(1783) = 3 \cdot 829 \cdot 1279;\ \Phi_5(1783) = 31 \cdot 67271 \cdot 4849081$.

C, $31, 331, 5233, 42073^*, 193, 1783, 1279:\ \Phi_3(1279) = 3 \cdot 229 \cdot 2383;\ \Phi_7(1279) = 56701 \cdot 3745631 \cdot 20627531$.

C, $31, 331, 5233, 42073^*, 193, 1783, 1279, 2383:\ \Phi_3(2383) = 3 \cdot 151 \cdot 12541;$
$\Phi_7(2383) = 475637 \cdot 6770429 \cdot 56889841$.

C, $31, 331, 5233, 42073^*, 193, 1783, 1279, 2383, 151:$ contradiction to B.

C, $31, 331, 5233, 42073^*, 193, 1783, 1279, 2383, 475637:$ 475637 is inadmissible.

C, $31, 331, 5233, 42073^*, 193, 1783, 1279, 20627531:$ 20627531 is inadmissible.

C, $31, 331, 5233, 42073^{*}, 193, 1783, 4849081$: 4849081 is inadmissible.
C, $31, 331, 5233, 42073, 75931$: $\Phi_3(75931) = 3 \cdot 7^2 19 \cdot 43 \cdot 61 \cdot 787$.
C, $31, 331, 5233, 42073, 75931, 787$: $\Phi_3(787) = 3 \cdot 37^2 151$; $\Phi_5(787) = 570821 \cdot 672901$.
C, $31, 331, 5233, 42073, 75931, 787, 151$: contradiction to B.
C, $31, 331, 5233, 42073, 75931, 787, 672901$: $\Phi_2(672901) = 2 \cdot 157 \cdot 2143$.
C, $31, 331, 5233, 42073, 75931, 787, 672901^{*}, 2143$: $\Phi_3(2143) = 3 \cdot 43 \cdot 35617$.
C, $31, 331, 5233, 42073, 75931, 787, 672901^{*}, 2143, 35617$: $\Phi_3(35617) = 3 \cdot 19 \cdot 22256251$.
C, $31, 331, 5233, 42073, 75931, 787, 672901^{*}, 2143, 35617, 22256251$: $\Phi_3(22256251) = 3 \cdot 79 \cdot 397 \cdot 15667 \cdot 336031$.
C, $31, 331, 5233, 42073, 75931, 787, 672901^{*}, 2143, 35617, 22256251, 336031$: 336031 is inadmissible.
C, $31, 331, 5233, 41213191$: $\Phi_3(41213191) = 3 \cdot 199 \cdot 3217 \cdot 4831 \cdot 183067$.
C, $31, 331, 5233, 41213191, 183067$: $\Phi_3(183067) = 3 \cdot 1117 \cdot 10001107$.
C, $31, 331, 5233, 41213191, 183067, 10001107$: $\Phi_3(10001107) = 3 \cdot 7 \cdot 79 \cdot 42409 \cdot 1421647$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409$: $\Phi_2(42409) = 2 \cdot 5 \cdot 4241$;
$\Phi_3(42409) = 3 \cdot 13 \cdot 223 \cdot 206803$; $\Phi_5(42409) = 11 \cdot 491 \cdot 3571 \cdot 12611 \cdot 13299301$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409^{*}, 4241$: $\Phi_3(4241) = 13 \cdot 31 \cdot 44641$;
$\Phi_7(4241) = 29 \cdot 197 \cdot 137957 \cdot 463303 \cdot 15938189$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409^{*}, 4241, 44641$: $\Phi_3(44641) = 3 \cdot 7 \cdot 94898263$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409^{*}, 4241, 44641, 94898263$: 94898263 is inadmissible.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409^{*}, 4241, 463303$: $\Phi_3(463303) = 3 \cdot 13 \cdot 19 \cdot 37 \cdot 37 \cdot 211597$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409^{*}, 4241, 463303, 211597$: 211597 is inadmissible.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 206803$: $\Phi_3(206803) = 3 \cdot 7 \cdot 19 \cdot 37 \cdot 61 \cdot 47491$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 206803, 47491$: $\Phi_3(47491) = 3 \cdot 163 \cdot 1063 \cdot 4339$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 206803, 47491, 1063$: $\Phi_3(1063) = 3 \cdot 377011$;
$\Phi_7(1063) = 337 \cdot 2423 \cdot 1289513 \cdot 1371511$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 206803, 47491, 1063, 377011$: 377011 is inadmissible.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 206803, 47491, 1063, 1371511$:
$\Phi_3(1371511) = 3 \cdot 1579 \cdot 1759 \cdot 225751$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 206803, 47491, 1063, 1371511, 225751$:
$\Phi_3(225751) = 3 \cdot 5443 \cdot 3121057$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 206803, 47491, 1063, 1371511, 225751, 5443$:
$\Phi_3(5443) = 3 \cdot 7 \cdot 13 \cdot 108541$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 206803, 47491, 1063, 1371511, 225751, 5443, 108541$:
$\Phi_2(108541) = 2 \cdot 7 \cdot 7753$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 206803, 47491, 1063, 1371511, 225751, 5443, 108541^{*}$,
7753 : $\Phi_3(7753) = 3 \cdot 7 \cdot 2862703$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 206803, 47491, 1063, 1371511, 225751, 5443, 108541^{*}$,
$7753, 2862703$: 2862703 is inadmissible.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 13299301$: $\Phi_2(13299301) = 2 \cdot 283 \cdot 23497$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 13299301^{*}, 23497$: $\Phi_3(23497) = 3 \cdot 73 \cdot 2521153$.
C, $31, 331, 5233, 41213191, 183067, 10001107, 42409, 13299301^{*}, 23497, 2521153$: 2521153 is inadmissible.
C, $31, 331, 63601$: $\Phi_2(63601) = 2 \cdot 7^2 11 \cdot 59$; $\Phi_3(63601) = 3 \cdot 2203 \cdot 612067$;
$\Phi_5(63601) = 5 \cdot 41 \cdot 271 \cdot 1381 \cdot 4231 \cdot 50408381$.
C, $31, 331, 63601^{*}, 37861$: $\Phi_3(37861) = 3 \cdot 37 \cdot 1201 \cdot 10753$.
(Remark: 37861 is the divisor of $\Phi_5(331)$ which appeared in the third line of C. We often use this method.)
C, $31, 331, 63601^{*}, 37861, 10753$: $\Phi_3(10753) = 3 \cdot 151 \cdot 397 \cdot 643$.
C, $31, 331, 63601^{*}, 37861, 10753, 151$: contradiction to B.

$C, 31, 331, 63601, 612067:\ \Phi_3(612067) = 3 \cdot 4801 \cdot 26010319.$

$C, 31, 331, 63601, 612067, 26010319:\ 26010319$ is inadmissible.

$C, 31, 331, 63601, 50408381:\ \Phi_2(50408381) = 2 \cdot 3 \cdot 8401397.$

$C, 31, 331, 63601, 50408381^*, 8401397:\ \Phi_3(8401397) = 7 \cdot 31 \cdot 3943 \cdot 82492897.$

$C, 31, 331, 63601, 50408381^*, 8401397, 82492897:\ 82492897$ is inadmissible.

$C, 31, 17351:\ \Phi_3(17351) = 13 \cdot 1063 \cdot 21787.$

$C, 31, 17351, 21787:\ \Phi_3(21787) = 3 \cdot 31 \cdot 5104249.$

$C, 31, 17351, 21787, 5104249:\ \Phi_2(5104249) = 2 \cdot 5^3 17 \cdot 1201;\ \Phi_3(5104249) = 3 \cdot 61 \cdot 216781 \cdot 656737.$

$C, 31, 17351, 21787, 5104249^*, 1201:\ \Phi_3(1201) = 3 \cdot 7 \cdot 68743.$

$C, 31, 17351, 21787, 5104249^*, 1201, 68743:\ 68743$ is inadmissible.

$C, 31, 17351, 21787, 5104249, 656737:\ \Phi_2(656737) = 2 \cdot 41 \cdot 8009;\ \Phi_3(656737) = 3 \cdot 7 \cdot 13 \cdot 661 \cdot 787 \cdot 3037.$

$C, 31, 17351, 21787, 5104249, 656737^*, 8009:\ \Phi_3(8009) = 6661 \cdot 9631;$
$\Phi_7(8009) = 7 \cdot 43 \cdot 127 \cdot 491 \cdot 127247 \cdot 305873 \cdot 361313.$

$C, 31, 17351, 21787, 5104249, 656737^*, 8009, 9631:\ \Phi_3(9631) = 3 \cdot 151 \cdot 204781.$

$C, 31, 17351, 21787, 5104249, 656737^*, 8009, 9631, 151:$ contradiction to B.

$C, 31, 17351, 21787, 5104249, 656737^*, 8009, 305873:\ 305873$ is inadmissible.

$C, 31, 17351, 21787, 5104249, 656737, 787:\ \Phi_3(787) = 3 \cdot 37 \cdot 37 \cdot 151;\ \Phi_5(787) = 570821 \cdot 672901.$

$C, 31, 17351, 21787, 5104249, 656737, 787, 151:$ contradiction to B.

$C, 31, 17351, 21787, 5104249, 656737, 787, 570821:\ \Phi_2(570821) = 2 \cdot 3 \cdot 7 \cdot 13591.$

$C, 31, 17351, 21787, 5104249, 656737, 787, 570821^*, 13591:\ \Phi_3(13591) = 3 \cdot 7 \cdot 7 \cdot 1256659.$

$C, 31, 17351, 21787, 5104249, 656737, 787, 570821^*, 13591, 1256659:\ \Phi_3(1256659) = 3 \cdot 8599 \cdot 61216153.$

$C, 31, 17351, 21787, 5104249, 656737, 787, 570821^*, 13591, 1256659, 61216153:\ 61216153$ is inadmissible.

$C, 31, 42407:\ \Phi_3(42407) = 1471 \cdot 1222567.$

$C, 31, 42407, 1222567:\ 1222567$ is inadmissible.

D. $127 \nmid n$.

$D, 127:\ \Phi_3(127) = 3 \cdot 5419;\ \Phi_7(127) = 7 \cdot 43 \cdot 86353 \cdot 162709.$

$D, 127, 5419:\ \Phi_3(5419) = 3 \cdot 31 \cdot 313 \cdot 1009.$

$D, 127, 5419, 31:$ contradiction to C.

$D, 127, 162709:\ \Phi_2(162709) = 2 \cdot 5 \cdot 53 \cdot 307.$

$D, 127, 162709^*, 86353:\ \Phi_5(86353) = 11 \cdot 281 \cdot 1021 \cdot 1964041 \cdot 8970971.$

$D, 127, 162709^*, 86353, 8970971:\ 8970971$ is inadmissible.

E. $19 \nmid n$.

$E, 19:\ \Phi_3(19) = 3 \cdot 127;\ \Phi_5(19) = 151 \cdot 911;\ \Phi_7(19) = 701 \cdot 70841;\ \Phi_{11}(19) = 104281 \cdot 62060021.$

$E, 19, 127:$ contradiction to D.

$E, 19, 151:$ contradiction to B.

$E, 19, 70841:\ \Phi_2(70841) = 2 \cdot 3 \cdot 11807;\ \Phi_3(70841) = 39103 \cdot 128341.$

$E, 19, 70841^*, 701:\ \Phi_3(701) = 492103.$

$E, 19, 70841^*, 701, 492103:\ \Phi_3(492103) = 3 \cdot 307 \cdot 1609 \cdot 163417.$

$E, 19, 70841^*, 701, 492103, 163417:\ \Phi_3(163417) = 3 \cdot 7 \cdot 463 \cdot 2746609.$

$E, 19, 70841^*, 701, 492103, 163417, 2746609:\ \Phi_3(2746609) = 3 \cdot 19 \cdot 127 \cdot 1879 \cdot 554611.$

$E, 19, 70841^*, 701, 492103, 163417, 2746609, 127:$ contradiction to D.

$E, 19, 70841, 39103:\ \Phi_3(39103) = 3 \cdot 7561 \cdot 67411.$

$E, 19, 70841, 39103, 67411:\ 67411$ is inadmissible.

$E, 19, 62060021:\ \Phi_2(62060021) = 2 \cdot 3^2 47 \cdot 109 \cdot 673.$

$E, 19, 62060021^*, 104281:\ \Phi_3(104281) = 3 \cdot 7 \cdot 43 \cdot 67 \cdot 179743.$

$E, 19, 62060021^*, 104281, 179743:\ \Phi_3(179743) = 3 \cdot 7 \cdot 31 \cdot 49627843.$

E, $19, 62060021^*, 104281, 179743, 31:$ contradiction to C.

F. $11 \nmid n$.
F, $11:$ $\Phi_3(11) = 7 \cdot 19$; $\Phi_5(11) = 5 \cdot 3221$; $\Phi_7(11) = 43 \cdot 45319$; $\Phi_{11}(11) = 15797 \cdot 1806113$.
F, $11, 19:$ contradiction to E.
F, $11, 3221:$ $\Phi_2(3221) = 2 \cdot 3^2 \cdot 179$; $\Phi_3(3221) = 10378063$; $\Phi_7(3221) = 7 \cdot 673 \cdot 10333 \cdot 248879 \cdot 92204351$.
F, $11, 3221^*, 179:$ $\Phi_3(179) = 7 \cdot 4603$; $\Phi_5(179) = 11 \cdot 93853931$.
F, $11, 3221^*, 179, 4603:$ $\Phi_3(4603) = 3 \cdot 7 \cdot 1009153$; $\Phi_3(4603) = 11 \cdot 911 \cdot 208511 \cdot 214891$.
F, $11, 3221^*, 179, 4603, 1009153:$ 1009153 is inadmissible.
F, $11, 3221^*, 179, 4603, 208511:$ $\Phi_3(208511) = 7 \cdot 31 \cdot 61 \cdot 661 \cdot 4969$.
F, $11, 3221^*, 179, 4603, 208511, 31:$ contradiction to C.
F, $11, 3221^*, 179, 93853931:$ $\Phi_3(93853931) = 19^2 \cdot 31 \cdot 151 \cdot 8293 \cdot 628561$.
F, $11, 3221^*, 179, 93853931, 151:$ contradiction to B.
F, $11, 3221, 10378063:$ 10378063 is inadmissible.
F, $11, 3221, 248879:$ 248879 is inadmissible.
F, $11, 45319:$ $\Phi_3(45319) = 3 \cdot 127 \cdot 5390701$.
F, $11, 45319, 127:$ contradiction to D.
F, $11, 1806113:$ $\Phi_2(1806113) = 2 \cdot 3 \cdot 17 \cdot 17707$.
F, $11, 1806113^*, 17707:$ $\Phi_3(17707) = 3 \cdot 7^2 2133031$.
F, $11, 1806113^*, 17707, 2133031:$ 2133031 is inadmissible.

G. $7 \nmid n$.
G, $7:$ $\Phi_3(7) = 3 \cdot 19$; $\Phi_5(7) = 2801$; $\Phi_7(7) = 29 \cdot 4733$; $\Phi_{11}(7) = 1123 \cdot 293459$.
G, $7, 19:$ contradiction to E.
G, $7, 2801:$ $\Phi_2(2801) = 2 \cdot 3 \cdot 467$; $\Phi_3(2801) = 37 \cdot 43 \cdot 4933$; $\Phi_5(2801) = 5 \cdot 1956611 \cdot 6294091$.
G, $7, 2801^*, 467:$ $\Phi_3(467) = 19 \cdot 11503$; $\Phi_5(467) = 11 \cdot 31 \cdot 41 \cdot 3409261$.
G, $7, 2801^*, 467, 19:$ contradiction to E.
G, $7, 2801^*, 467, 31:$ contradiction to C.
G, $7, 2801, 4933:$ $\Phi_2(4933) = 2 \cdot 2467$; $\Phi_3(4933) = 3 \cdot 127 \cdot 193 \cdot 331$.
G, $7, 2801, 4933^*, 2467:$ $\Phi_3(2467) = 3 \cdot 271 \cdot 7489$.
G, $7, 2801, 4933^*, 2467, 271:$ $\Phi_3(271) = 3 \cdot 24571$; $\Phi_5(271) = 5 \cdot 251 \cdot 4313591$; $\Phi_7(271) = 9170197 \cdot 43355341$.
G, $7, 2801, 4933^*, 2467, 271, 24571:$ 24571 is inadmissible.
G, $7, 2801, 4933^*, 2467, 271, 4313591:$ 4313591 is inadmissible.
G, $7, 2801, 4933^*, 2467, 271, 9170197:$ 9170197 is inadmissible.
G, $7, 2801, 4933, 127:$ contradiction to D.
G, $7, 2801, 6294091:$ 6294091 is inadmissible.
G, $7, 4733:$ $\Phi_2(4733) = 2 \cdot 3^2 263$; $\Phi_3(4733) = 22406023$.
G, $7, 4733^*, 263:$ $\Phi_3(263) = 7^2 13 \cdot 109$.
G, $7, 4733^*, 263, 109:$ $\Phi_3(109) = 3 \cdot 7 \cdot 571$; $\Phi_5(109) = 31 \cdot 191 \cdot 24061$; $\Phi_7(109) = 113 \cdot 281 \cdot 53306107$.
G, $7, 4733^*, 263, 109, 571:$ $\Phi_3(571) = 3 \cdot 7 \cdot 103 \cdot 151$; $\Phi_5(571) = 5 \cdot 1831 \cdot 11631811$.
G, $7, 4733^*, 263, 109, 571, 151:$ contradiction to B.
G, $7, 4733^*, 263, 109, 571, 11631811:$ $\Phi_3(11631811) = 3 \cdot 7 \cdot 13 \cdot 19 \cdot 37^2 73 \cdot 211 \cdot 1237$.
G, $7, 4733^*, 263, 109, 571, 11631811, 19:$ contradiction to E.
G, $7, 4733^*, 263, 109, 31:$ contradiction to C.
G, $7, 4733^*, 263, 109, 53306107:$ 53306107 is inadmissible.
G, $7, 4733, 22406023:$ 22406023 is inadmissible.
G, $7, 293459:$ 293459 is inadmissible.

H. $23 \nmid n$.
H, 23 : $\Phi_3(23) = 7 \cdot 79$; $\Phi_5(23) = 292561$; $\Phi_7(23) = 29 \cdot 5336717$.
H, 23, 7 : contradiction to G.
H, 23, 292561 : $\Phi_2(292561) = 2 \cdot 19 \cdot 7699$.
H, 23, 292561*, 19 : contradiction to E.
H, 23, 5336717 : $\Phi_2(5336717) = 2 \cdot 3 \cdot 889453$.
H, 23, 5336717*, 889453 : 889453 is inadmissible.

I. $131 \nmid n$.
I, 131 : $\Phi_3(131) = 17293$; $\Phi_5(131) = 5 \cdot 61 \cdot 973001$; $\Phi_7(131) = 127 \cdot 189967 \cdot 211093$.
I, 131, 17293 : $\Phi_2(17293) = 2 \cdot 8647$; $\Phi_3(17293) = 3 \cdot 13 \cdot 7668337$.
I, 131, 17293*, 8647 : $\Phi_3(8647) = 3 \cdot 7 \cdot 37 \cdot 157 \cdot 613$.
I, 131, 17293*, 8647, 7 : contradiction to G.
I, 131, 17293, 7668337 : $\Phi_2(7668337) = 2 \cdot 23 \cdot 166703$; $\Phi_3(7668337) = 3 \cdot 801337 \cdot 24460537$.
I, 131, 17293, 7668337*, 23 : contradiction to H.
I, 131, 17293, 7668337, 801337 : $\Phi_2(801337) = 2 \cdot 59 \cdot 6791$.
I, 131, 17293, 7668337, 801337*, 24460537 : 24460537 is inadmissible.
I, 131, 973001 : $\Phi_2(973001) = 2 \cdot 3 \cdot 257 \cdot 631$.
I, 131, 973001*, 257 : $\Phi_3(257) = 61 \cdot 1087$.
I, 131, 973001*, 257, 1087 : $\Phi_3(1087) = 3 \cdot 7 \cdot 199 \cdot 283$.
I, 131, 973001*, 257, 1087, 7 : contradiction to G.
I, 131, 127 : contradiction to D.

J. $37 \nmid n$.
J, 37 : $\Phi_2(37) = 2 \cdot 19$; $\Phi_3(37) = 3 \cdot 7 \cdot 67$; $\Phi_5(37) = 11 \cdot 41 \cdot 4271$; $\Phi_7(37) = 71 \cdot 37140797$.
J, 37*, 19 : contradiction to E
J, 37, 7 : contradiction to G.
J, 37, 11 : contradiction to F.
J, 37, 37140797 : $\Phi_2(37140797) = 2 \cdot 3 \cdot 347 \cdot 17839$; $\Phi_3(37140797) = 1609 \cdot 3469 \cdot 4831 \cdot 51157$.
J, 37, 37140797*, 17839 : $\Phi_3(17839) = 3 \cdot 13 \cdot 8160199$.
J, 37, 37140797*, 17839, 8160199 : 8160199 is inadmissible.
J, 37, 37140797, 3469 : $\Phi_2(3469) = 2 \cdot 5 \cdot 347$; $\Phi_3(3469) = 3 \cdot 7 \cdot 19 \cdot 30169$.
J, 37, 37140797, 3469*, 1609 : $\Phi_3(1609) = 3 \cdot 863497$.
J, 37, 37140797, 3469*, 1609, 863497 : 863497 is inadmissible.
J, 37, 37140797, 3469, 19 : contradiction to E.

K. $61 \nmid n$.
K, 61 : $\Phi_2(61) = 2 \cdot 31$; $\Phi_3(61) = 3 \cdot 13 \cdot 97$; $\Phi_5(61) = 5 \cdot 131 \cdot 21491$.
K, 61*, 31 : contradiction to C.
K, 61, 97 : $\Phi_2(97) = 2 \cdot 7^2$; $\Phi_3(97) = 3 \cdot 3169$; $\Phi_5(97) = 11 \cdot 31 \cdot 262321$; $\Phi_7(97) = 43 \cdot 967 \cdot 20241187$.
K, 61, 97*, 7 : contradiction to G.
K, 61, 97, 3169 : $\Phi_2(3169) = 2 \cdot 5 \cdot 317$; $\Phi_3(3169) = 3 \cdot 3348577$.
K, 61, 97, 3169*, 317 : $\Phi_3(317) = 7 \cdot 14401$; $\Phi_5(317) = 11 \cdot 311 \cdot 2961121$.
K, 61, 97, 3169*, 317, 7 : contradiction to G.
K, 61, 97, 3169*, 317, 11 : contradiction to F.
K, 61, 97, 3169, 3348577 : $\Phi_2(3348577) = 2 \cdot 1674289$.
K, 61, 97, 3169, 3348577*, 1674289 : 1674289 is inadmissible.
K, 61, 97, 11 : contradiction to F.

$K,\ 61, 97, 20241187:$ 20241187 is inadmissible.

$K,\ 61, 131:$ contradiction to I.

L. $13 \nmid n$.

$L,\ 13:$ $\Phi_2(13) = 2\cdot 7$; $\Phi_3(13) = 3\cdot 61$; $\Phi_5(13) = 30941$; $\Phi_7(13) = 5229043$;
$\Phi_{11}(13) = 23\cdot 419\cdot 859\cdot 18041$; $\Phi_{13}(13) = 53\cdot 264031\cdot 1803647$.

$L,\ 13^*, 7:$ contradiction to G.

$L,\ 13, 61:$ contradiction to K.

$L,\ 13, 30941:$ $\Phi_2(30941) = 2\cdot 3^4 191$; $\Phi_3(30941) = 157\cdot 433\cdot 14083$.

$L,\ 13, 30941^*, 191:$ $\Phi_3(191) = 7\cdot 13^2 31$; $\Phi_5(191) = 5\cdot 11\cdot 1871\cdot 13001$;
$\Phi_7(191) = 127\cdot 197\cdot 10627\cdot 183569$; $\Phi_{13}(191) = 131\cdot 1483\cdot 9049\cdot 92041\cdot 301627\cdot 48552947$.

$L,\ 13, 30941^*, 191, 7:$ contradiction to G.

$L,\ 13, 30941^*, 191, 11:$ contradiction to F.

$L,\ 13, 30941^*, 191, 127:$ contradiction to D.

$L,\ 13, 30941^*, 191, 131:$ contradiction to I.

$L,\ 13, 30941, 433:$ $\Phi_2(433) = 2\cdot 7\cdot 31$; $\Phi_3(433) = 3\cdot 37\cdot 1693$; $\Phi_5(433) = 11\cdot 1811\cdot 1768661$.

$L,\ 13, 30941, 433^*, 31:$ contradiction to C.

$L,\ 13, 30941, 433, 37:$ contradiction to J.

$L,\ 13, 30941, 433, 11:$ contradiction to F.

$L,\ 13, 5229043:$ $\Phi_3(5229043) = 3\cdot 31\cdot 4051\cdot 72577051$.

$L,\ 13, 5229043, 31:$ contradiction to C.

$L,\ 13, 23:$ contradiction to H.

$L,\ 13, 1803647:$ 1803647 is inadmissible.

M. $3 \nmid n$.

$M,\ 3:$ $\Phi_3(3) = 13$; $\Phi_5(3) = 11^2$; $\Phi_7(3) = 1093$; $\Phi_{11}(3) = 23\cdot 3851$; $\Phi_{13}(11) = 797161$;
$\Phi_{17}(3) = 1871\cdot 34511$; $\Phi_{19}(3) = 1597\cdot 363889$; $\Phi_{29}(3) = 59\cdot 28537\cdot 20381027$;
$\Phi_{31}(3) = 683\cdot 102673\cdot 4404047$; $\Phi_{47}(3) = 1223\cdot 21997\cdot 5112661\cdot 96656723$.

$M,\ 3, 13:$ contradiction to L.

$M,\ 3, 11:$ contradiction to F.

$M,\ 3, 1093:$ contradiction to A.

$M,\ 3, 23:$ contradiction to H.

$M,\ 3, 797161:$ $\Phi_2(797161) = 2\cdot 398581$; $\Phi_3(797161) = 3\cdot 61\cdot 151\cdot 22996651$.

$M,\ 3, 797161^*, 398581:$ $\Phi_3(398581) = 3\cdot 1621\cdot 32668561$.

$M,\ 3, 797161^*, 398581, 32668561:$ 32668561 is inadmissible.

$M,\ 3, 797161, 151:$ contradiction to B.

$M,\ 3, 1871:$ $\Phi_3(1871) = 7\cdot 157\cdot 3187$.

$M,\ 3, 1871, 7:$ contradiction to G.

$M,\ 3, 1597:$ $\Phi_2(1597) = 2\cdot 17\cdot 47$; $\Phi_3(1597) = 3\cdot 43\cdot 73\cdot 271$.

$M,\ 3, 1597^*, 47:$ $\Phi_3(47) = 37\cdot 61$; $\Phi_5(47) = 11\cdot 31\cdot 14621$; $\Phi_{13}(47) = 53\cdot 2237\cdot 14050609\cdot 71265169$.

$M,\ 3, 1597^*, 47, 37:$ contradiction to J.

$M,\ 3, 1597^*, 47, 31:$ contradiction to C.

$M,\ 3, 1597^*, 47, 14050609:$ 14050609 is inadmissible.

$M,\ 3, 1597, 271:$ $\Phi_3(271) = 3\cdot 24571$; $\Phi_5(271) = 5\cdot 251\cdot 4313591$; $\Phi_7(271) = 9170197\cdot 43355341$.

$M,\ 3, 1597, 271, 24571:$ 24571 is inadmissible.

$M,\ 3, 1597, 271, 4313591:$ 4313591 is inadmissible.

$M,\ 3, 1597, 271, 9170197:$ $\Phi_2(9170197) = 2\cdot 19\cdot 241321$.

$M,\ 3, 1597, 271, 9170197, 19:$ contradiction to E.

M, $3, 20381027:$ $\Phi_3(20381027) = 7 \cdot 67 \cdot 19687 \cdot 44988319$.

M, $3, 20381027, 7:$ contradiction to G.

M, $3, 683:$ $\Phi_3(683) = 7 \cdot 66739$.

M, $3, 683, 7:$ contradiction to G.

M, $3, 96656723:$ 96656723 is inadmissible.

N. $5 \nmid n$.

N, $5:$ $\Phi_2(5) = 2 \cdot 3$; $\Phi_3(5) = 31$; $\Phi_5(5) = 11 \cdot 71$; $\Phi_7(5) = 19531$; $\Phi_{11}(5) = 12207031$; $\Phi_{11}(5) = 191 \cdot 6271 \cdot 3981071$.

N, $5^*, 3:$ contradiction to M.

N, $5, 31:$ contradiction to C.

N, $5, 11:$ contradiction to F.

N, $5, 19531:$ $\Phi_5(19531) = 5 \cdot 191 \cdot 4760281 \cdot 32009891$.

N, $5, 19531, 32009891:$ $\Phi_3(32009891) = 7 \cdot 283 \cdot 468913 \cdot 1103041$.

N, $5, 19531, 32009891, 7:$ contradiction to G.

N, $5, 12207031:$ $\Phi_3(12207031) = 3 \cdot 7 \cdot 1041757 \cdot 6811369$.

N, $5, 12207031, 7:$ contradiction to G.

N, $5, 3981071:$ 3981071 is inadmissible.

O. $29 \nmid n$.

O, $29:$ $\Phi_2(29) = 2 \cdot 3 \cdot 5$; $\Phi_3(29) = 13 \cdot 67$; $\Phi_5(29) = 732541$; $\Phi_7(29) = 7 \cdot 88009573$.

O, $29^*, 3:$ contradiction to M.

O, $29, 13:$ contradiction to L.

O, $29, 732541:$ $\Phi_2(732541) = 2 \cdot 47 \cdot 7793$; $\Phi_3(732541) = 3 \cdot 43 \cdot 271 \cdot 15349897$.

O, $29, 732541^*, 7793:$ $\Phi_3(7793) = 7 \cdot 8676949$; $\Phi_3(7793) = 11 \cdot 71^2 3701 \cdot 17974051$.

O, $29, 732541^*, 7793, 7:$ contradiction to G.

O, $29, 732541^*, 7793, 11:$ contradiction to F.

O, $29, 732541, 3:$ contradiction to M.

O, $29, 7:$ contradiction to G.

P. $43 \nmid n$.

P, $43:$ $\Phi_3(43) = 3 \cdot 631$; $\Phi_5(43) = 3500201$; $\Phi_7(43) = 7 \cdot 5839 \cdot 158341$.

P, $43, 3:$ contradiction to M.

P, $43, 3500201:$ $\Phi_2(3500201) = 2 \cdot 3 \cdot 583367$; $\Phi_3(3500201) = 13 \cdot 139 \cdot 28411 \cdot 238639$.

P, $43, 3500201^*, 3:$ contradiction to M.

P, $43, 3500201, 13:$ contradiction to L.

P, $43, 7:$ contradiction to G.

Q. $1051 \nmid n$.

Q, $1051:$ $\Phi_3(1051) = 3 \cdot 368551$; $\Phi_5(1051) = 5 \cdot 71 \cdot 241 \cdot 14275091$.

Q, $1051, 3:$ contradiction to M.

Q, $1051, 5:$ contradiction to N.

R. $17 \nmid n$.

R, $17:$ $\Phi_2(17) = 2 \cdot 3^2$; $\Phi_3(17) = 307$; $\Phi_5(17) = 88741$; $\Phi_7(17) = 25646167$.

R, $17^*, 3:$ contradiction to M.

R, $17, 307$; $\Phi_3(307) = 3 \cdot 43 \cdot 733$; $\Phi_5(307) = 1051 \cdot 1621 \cdot 5231$.

R, $17, 307, 3:$ contradiction to M.

R, $17, 307, 1051:$ contradiction to Q.

R, $17, 88741:$ $\Phi_2(88741) = 2 \cdot 44371$; $\Phi_3(88741) = 5 \cdot 71 \cdot 241 \cdot 14275091$.

R, $17, 88741^*, 44371$: 44371 is inadmissible.
R, $17, 88741, 5$: contradiction to N.
R, $17, 25646167$: 25646167 is inadmissible.

S. $71 \nmid n$.
S, 71 : $\Phi_3(71) = 5113$; $\Phi_5(71) = 5 \cdot 11 \cdot 211 \cdot 2221$; $\Phi_7(71) = 7 \cdot 883 \cdot 21020917$.
S, $71, 5113$: $\Phi_2(5113) = 2 \cdot 2557$; $\Phi_3(5113) = 3 \cdot 8715961$.
S, $71, 5113^*, 2557$: $\Phi_3(2557) = 3 \cdot 7 \cdot 13^2 19 \cdot 97$.
S, $71, 5113^*, 2557, 19$: contradiction to E.
S, $71, 5113, 3$: contradiction to M.
S, $71, 11$: contradiction to F.
S, $71, 7$: contradiction to G.

T. $113 \nmid n$.
T, 113 : $\Phi_2(113) = 2 \cdot 3 \cdot 19$; $\Phi_3(113) = 13 \cdot 991$; $\Phi_5(113) = 11 \cdot 251 \cdot 59581$; $\Phi_7(113) = 7 \cdot 44983 \cdot 6670903$.
T, $113^*, 19$: contradiction to E.
T, $113, 13$: contradiction to L.
T, $113, 11$: contradiction to F.
T, $113, 7$: contradiction to G.

U. $197 \nmid n$.
U, 197 : $\Phi_2(197) = 2 \cdot 3^2 11$; $\Phi_3(197) = 19 \cdot 2053$; $\Phi_5(197) = 661 \cdot 991 \cdot 2311$; $\Phi_7(197) = 7 \cdot 29 \cdot 97847 \cdot 2957767$.
U, $197^*, 11$: contradiction to F.
U, $197, 19$: contradiction to E.
U, $197, 991$: $\Phi_3(991) = 3 \cdot 7 \cdot 13^2 277$.
U, $197, 991, 7$: contradiction to G.
U, $197, 7$: contradiction to G.

V. $211 \nmid n$.
V, 211 : $\Phi_3(211) = 3 \cdot 13 \cdot 31 \cdot 37$; $\Phi_5(211) = 5 \cdot 1361 \cdot 292661$; $\Phi_7(211) = 7 \cdot 307189 \cdot 41233879$.
V, $211, 31$: contradiction to C.
V, $211, 5$: contradiction to N.
V, $211, 7$: contradiction to G.

W. $239 \nmid n$.
W, 239 : $\Phi_3(239) = 19 \cdot 3019$; $\Phi_7(239) = 7 \cdot 29 \cdot 245561 \cdot 3754507$.
W, $239, 19$: contradiction to E.
W, $239, 7$: contradiction to G.

X. $281 \nmid n$.
X, 281 : $\Phi_2(281) = 2 \cdot 3 \cdot 47$; $\Phi_3(281) = 109 \cdot 727$; $\Phi_5(281) = 5 \cdot 31 \cdot 271 \cdot 148961$.
X, $281^*, 3$: contradiction to M.
X, $281, 727$: $\Phi_3(727) = 3 \cdot 176419$; $\Phi_5(727) = 14281 \cdot 19587401$.
X, $281, 727, 3$: contradiction to M.
X, $281, 727, 19587401$: $\Phi_2(19587401) = 2 \cdot 3^2 311 \cdot 3499$.
X, $281, 727, 19587401^*, 3$: contradiction to M.
X, $281, 31$: contradiction to C.

Y. $337 \nmid n$.
Y, 337 : $\Phi_2(337) = 2 \cdot 13^2$; $\Phi_3(337) = 3 \cdot 43 \cdot 883$.
Y, $337^*, 13$: contradiction to L.

Y, $337, 3:$ contradiction to M.

Z. $379 \nmid n$.
Z, $379:$ $\Phi_3(379) = 3 \cdot 61 \cdot 787$; $\Phi_5(379) = 11 \cdot 41 \cdot 45869891$.
Z, $379, 61:$ contradiction to K.
Z, $379, 11:$ contradiction to F.

AA. $421 \nmid n$.
AA, $421:$ $\Phi_2(421) = 2 \cdot 211$; $\Phi_3(421) = 3 \cdot 59221$; $\Phi_5(421) = 5 \cdot 11 \cdot 181 \cdot 191 \cdot 16561$.
AA, $421^*, 211:$ contradiction to V.
AA, $421, 3:$ contradiction to M.
AA, $421, 11:$ contradiction to F.

AB. $449 \nmid n$.
AB, $449:$ $\Phi_2(449) = 2 \cdot 3^2 5^2$; $\Phi_3(449) = 97 \cdot 2083$.
AB, $449^*, 5:$ contradiction to N.
AB, $449, 2083:$ $\Phi_3(2083) = 3 \cdot 7 \cdot 13 \cdot 15901$.
AB, $449, 2083, 7:$ contradiction to G.

AC. $463 \nmid n$.
AC, $463:$ $\Phi_3(463) = 3 \cdot 19 \cdot 3769$; $\Phi_5(463) = 881 \cdot 52274161$.
AC, $463, 19:$ contradiction to E.
AC, $463, 881:$ $\Phi_2(881) = 2 \cdot 3^2 7^2$; $\Phi_3(881) = 19 \cdot 40897$; $\Phi_5(881) = 5 \cdot 146521 \cdot 823241$.
AC, $463, 881^*, 7:$ contradiction to G.
AC, $463, 881, 19:$ contradiction to E.
AC, $463, 881, 5:$ contradiction to N.

AD. $491 \nmid n$.
AD, $491:$ $\Phi_3(491) = 37 \cdot 6529$; $\Phi_5(491) = 5 \cdot 101 \cdot 191 \cdot 603791$.
AD, $491, 37:$ contradiction to J.
AD, $491, 5:$ contradiction to N.

AE. $547 \nmid n$.
AE, $547:$ $\Phi_3(547) = 3 \cdot 163 \cdot 613$.
AE, $547, 3:$ contradiction to M.

AF. $617 \nmid n$.
AF, $617:$ $\Phi_2(617) = 2 \cdot 3 \cdot 103$; $\Phi_3(617) = 97 \cdot 3931$.
AF, $617^*, 3:$ contradiction to M.
AF, $617, 3931:$ $\Phi_3(3931) = 3 \cdot 7 \cdot 31 \cdot 23743$.
AF, $617, 3931, 31:$ contradiction to C.

AG. $631 \nmid n$.
AG, $631:$ $\Phi_3(631) = 3 \cdot 307 \cdot 433$; $\Phi_5(631) = 5 \cdot 11 \cdot 41 \cdot 1511 \cdot 46601$.
AG, $631, 3:$ contradiction to M.
AG, $631, 11:$ contradiction to F.

AH. $659 \nmid n$.
AH, $659:$ $\Phi_3(659) = 13 \cdot 33457$; $\Phi_5(659) = 31 \cdot 6131 \cdot 993821$.
AH, $659, 13:$ contradiction to L.
AH, $659, 31:$ contradiction to C.

AI. $673 \nmid n$.

AI, $673: \ \Phi_2(673) = 2 \cdot 337;\ \Phi_3(673) = 3 \cdot 151201;\ \Phi_5(673) = 421 \cdot 2531 \cdot 192811.$

AI, $673^*, 337:$ contradiction to Y.

AI, $673, 3:$ contradiction to M.

AI, $673, 421:$ contradiction to AA.

AJ. $743 \nmid n$.

AJ, $743: \ \Phi_3(743) = 552793.$

AJ, $743, 552793: \ \Phi_2(552793) = 2 \cdot 11 \cdot 25127;\ \Phi_3(552793) = 3 \cdot 19 \cdot 421 \cdot 12734119;$
$\Phi_5(552793) = 191 \cdot 661 \cdot 1531 \cdot 13931 \cdot 65521 \cdot 529271.$

AJ, $743, 552793^*, 11:$ contradiction to F.

AJ, $743, 552793, 19:$ contradiction to E.

AJ, $743, 552793, 13931:$ 13931 is inadmissible.

AK. $757 \nmid n$.

AK, $757: \ \Phi_2(757) = 2 \cdot 379;\ \Phi_3(757) = 3 \cdot 13 \cdot 14713;\ \Phi_5(757) = 11 \cdot 191 \cdot 2521 \cdot 62081.$

AK, $757^*, 379:$ contradiction to Z.

AK, $757, 13:$ contradiction to L.

AK, $757, 11:$ contradiction to F.

AL. $827 \nmid n$.

AL, $827: \ \Phi_3(827) = 684757.$

AL, $827, 684757: \ \Phi_2(684757) = 2 \cdot 342379.$

AL, $827, 684757^*, 342379: \ \Phi_3(342379) = 3 \cdot 7 \cdot 61 \cdot 2803 \cdot 32647.$

AL, $827, 684757^*, 342379, 7:$ contradiction to G.

AM. $911 \nmid n$.

AM, $911: \ \Phi_3(911) = 830833;\ \Phi_5(911) = 5 \cdot 11 \cdot 701 \cdot 17884211.$

AM, $911, 830833: \ \Phi_2(830833) = 2 \cdot 127 \cdot 3271;\ \Phi_3(830833) = 3 \cdot 13 \cdot 61 \cdot 337 \cdot 861001.$

AM, $911, 830833^*, 127:$ contradiction to D.

AM, $911, 830833, 61:$ contradiction to K.

AM, $911, 11:$ contradiction to F.

AN. $953 \nmid n$.

AN, $953: \ \Phi_2(953) = 2 \cdot 3^2 53;\ \Phi_3(953) = 181 \cdot 5023;\ \Phi_5(953) = 41 \cdot 1601 \cdot 2161 \cdot 5821;$
$\Phi_7(953) = 7 \cdot 29 \cdot 71 \cdot 113 \cdot 127 \cdot 379 \cdot 9566159.$

AN, $953^*, 3:$ contradiction to M.

AN, $953, 5023: \ \Phi_3(5023) = 3 \cdot 7 \cdot 19 \cdot 63247.$

AN, $953, 5023, 19:$ contradiction to E.

AN, $953, 2161: \ \Phi_2(2161) = 2 \cdot 23 \cdot 47;\ \Phi_3(2161) = 3 \cdot 13 \cdot 119797.$

AN, $953, 2161^*, 23:$ contradiction to H.

AN, $953, 2161, 13:$ contradiction to L.

AN, $953, 7:$ contradiction to G.

AO. $967 \nmid n$.

AO, $967: \ \Phi_3(967) = 3 \cdot 67 \cdot 4657.$

AO, $967, 3:$ contradiction to M.

A.4 Proof of Lemma 3.4

In order to prove Lemma 3.4, we need to show that (2.1) does not contain cyclotomic numbers described in §3.3. There are 84 such cyclotomic numbers. The writing method is similar to one of §A.3. In the proof, X is the set given in the statement of Lemma 3.3.

AP. 67 : $\Phi_7(67) = 175897 \cdot 522061$.
AP, $67, 522061$: $\Phi_2(522061) = 2 \cdot 261031$.
AP, $67, 522061^*, 261031$: 261031 is inadmissible.

AQ. 173 : $\Phi_7(173) = 3144079 \cdot 8576317$.
AQ, $173, 3144079$: $\Phi_3(3144079) = 3 \cdot 13 \cdot 67 \cdot 13267 \cdot 285151$.
AQ, $173, 3144079, 3$: $3 \in X$.

AR. 271 : $\Phi_7(271) = 9170197 \cdot 43355341$.
AR, $271, 9170197$: $\Phi_2(9170197) = 2 \cdot 19 \cdot 241321$.
AR, $271, 9170197^*, 19$: $19 \in X$.

AS. 347 : $\Phi_7(347) = 39577763 \cdot 44236319$.
AS, $347, 39577763$: 39577763 is inadmissible.

AT. 409 : $\Phi_7(409) = 6133 \cdot 15919 \cdot 48063373$.
AT, $409, 48063373$: $\Phi_2(48063373) = 2 \cdot 24031687$.
AT, $409, 48063373^*, 24031687$: 24031687 is inadmissible.

AU. 439 : $\Phi_7(439) = 7883 \cdot 63841 \cdot 14255627$.
AU, $439, 14255627$: $\Phi_3(14255627) = 13 \cdot 67 \cdot 811 \cdot 10501 \cdot 27397$.
AU, $439, 14255627, 13$: $13 \in X$.

AV. 607 : $\Phi_7(607) = 54517 \cdot 415759 \cdot 2210419$.
AV, $607, 415759$: 415759 is inadmissible.

AW. 619 : $\Phi_7(619) = 3389 \cdot 3732919 \cdot 4453751$.
AW, $619, 3732919$: $\Phi_3(3732919) = 3 \cdot 37 \cdot 73 \cdot 3673 \cdot 468199$.
AW, $619, 3732919, 3$: $3 \in X$.

AX. 653 : $\Phi_7(653) = 21757 \cdot 706763 \cdot 5049773$.
AX, $653, 21757$: $\Phi_2(21757) = 2 \cdot 11 \cdot 23 \cdot 43$.
AX, $653, 21757^*, 11$: $11 \in X$.

AY. 853 : $\Phi_7(853) = 2647 \cdot 11824121 \cdot 12321989$.
AY, $853, 11824121$: $\Phi_2(11824121) = 2 \cdot 3 \cdot 1381 \cdot 1427$.
AY, $853, 11824121^*, 3$: $3 \in X$.

AZ. 887 : $\Phi_7(887) = 5167 \cdot 6271651 \cdot 15045661$.
AZ, $887, 5167$: $\Phi_3(5167) = 3 \cdot 8901019$.
AZ, $887, 5167, 3$: $3 \in X$.

BA. 1279 : $\Phi_7(1279) = 56701 \cdot 3745631 \cdot 20627531$.
BA, $1279, 3745631$: $\Phi_3(3745631) = 31 \cdot 37 \cdot 73063 \cdot 167413$.
BA, $1279, 3745631, 31$: $31 \in X$.

BB. 1451 : $\Phi_7(1451) = 2381 \cdot 52584967 \cdot 74590391$.
BB, $1451, 52584967$: 52584967 is inadmissible.

BC. 2383 : $\Phi_7(2383) = 475637 \cdot 6770429 \cdot 56889841$.

BC, $2383, 475637$: $\Phi_2(475637) = 2 \cdot 3 \cdot 79273$.

BC, $2383, 475637^*, 3$: $3 \in X$.

BD. 3089 : $\Phi_7(3089) = 1303 \cdot 89237 \cdot 316793 \cdot 23592997$.

BD, $3089, 89237$: $\Phi_2(89237) = 2 \cdot 3 \cdot 107 \cdot 139$.

BD, $3089, 89237^*, 3$: $3 \in X$.

BE. 4129 : $\Phi_7(4129) = 5867 \cdot 17053 \cdot 714463 \cdot 69339047$.

BE, $4129, 69339047$: 69339047 is inadmissible.

BF. 4289 : $\Phi_7(4289) = 1471 \cdot 8807 \cdot 9619 \cdot 32467 \cdot 1538951$.

BF, $4289, 9619$: $\Phi_3(9619) = 3 \cdot 30844927$.

BF, $4289, 9619, 3$: $3 \in X$.

BG. 5399 : $\Phi_7(5399) = 2731 \cdot 9941 \cdot 14811889 \cdot 61602479$.

BG, $5399, 2731$: $\Phi_3(2731) = 3 \cdot 61 \cdot 40771$.

BG, $5399, 2731, 3$: $3 \in X$.

BH. 5689 : $\Phi_7(5689) = 9479 \cdot 107941 \cdot 338773 \cdot 97821473$.

BH, $5689, 107941$: $\Phi_2(107941) = 2 \cdot 31 \cdot 1741$; $\Phi_3(107941) = 3 \cdot 4111 \cdot 944731$.

BH, $5689, 107941^*, 31$: $31 \in X$.

BH, $5689, 107941, 3$: $3 \in X$.

BI. 5953 : $\Phi_7(5953) = 4663 \cdot 1352107 \cdot 1591927 \cdot 4434949$.

BI, $5953, 1591927$: 1591927 is inadmissible.

BJ. 10889 : $\Phi_7(10889) = 2003 \cdot 22093 \cdot 116341 \cdot 471997 \cdot 686057$.

BJ, $10889, 471997$: $\Phi_2(471997) = 2 \cdot 19 \cdot 12421$.

BJ, $10889, 471997^*, 19$: $19 \in X$.

BK. 17609 : $\Phi_7(17609) = 3109 \cdot 15289 \cdot 116747 \cdot 347873 \cdot 15444241$.

BK, $17609, 116747$: 116747 is inadmissible.

BL. 26833 : $\Phi_7(26833) = 26041 \cdot 11780777 \cdot 19965779 \cdot 60941581$.

BL, $26833, 19965779$: 19965779 is inadmissible.

BM. 48311 : $\Phi_7(48311) = 6751571 \cdot 7550173 \cdot 8327677 \cdot 29950187$.

BM, $48311, 7550173$: $\Phi_2(7550173) = 2 \cdot 31 \cdot 47 \cdot 2591$.

BM, $48311, 7550173^*, 31$: $31 \in X$.

BN. 56431 : $\Phi_7(56431) = 11047 \cdot 187573 \cdot 597367 \cdot 1912541 \cdot 13641041$.

BN, $56431, 11047, 597367$: $\Phi_3(597367) = 3 \cdot 19 \cdot 73 \cdot 85760137$.

BN, $56431, 11047, 597367, 3$: $3 \in X$.

BO. 63587 : $\Phi_7(63587) = 13063 \cdot 380059 \cdot 1341257 \cdot 1632751 \cdot 6079823$.

BO, $63587, 13063$: $\Phi_3(13063) = 3 \cdot 56885011$.

BO, $63587, 13063, 3$: $3 \in X$.

BP. 71209 : $\Phi_7(71209) = 46439 \cdot 128969 \cdot 160651 \cdot 10145059 \cdot 13357009$.

BP, $71209, 160651$: 160651 is inadmissible.

BQ. 109793 : $\Phi_7(109793) = 2731 \cdot 6287 \cdot 46831 \cdot 314581 \cdot 950111 \cdot 7288639$.

BQ, $109793, 2731$: $\Phi_3(2731) = 3 \cdot 61 \cdot 40771$.

BQ, $109793, 2731, 61$: $61 \in X$.

BR. 113287 : $\Phi_7(113287) = 2339 \cdot 3319 \cdot 5419 \cdot 2085931 \cdot 4027927 \cdot 5980619$.

BR, $113287, 5419:$ $\Phi_3(5419) = 3 \cdot 31 \cdot 313 \cdot 1009$.
BR, $113287, 5419, 3:$ $3 \in X$.

BS. $140009:$ $\Phi_7(140009) = 35869 \cdot 545651 \cdot 1282681 \cdot 9028013 \cdot 33234853$.
BS, $140009, 9028013:$ $\Phi_2(9028013) = 2 \cdot 3 \cdot 1504669$.
BS, $140009, 9028013^*, 3:$ $3 \in X$.

BT. $187049:$ $\Phi_7(187049) = 351779 \cdot 511463 \cdot 1506653 \cdot 5043011 \cdot 31329061$.
BT, $187049, 5043011:$ 5043011 is inadmissible.

BU. $191693:$ $\Phi_7(191693) = 7561 \cdot 11887 \cdot 14869 \cdot 16759 \cdot 89839 \cdot 118399 \cdot 208279$.
BU, $191693, 118399:$ 118399 is inadmissible.

BV. $262351:$ $\Phi_7(262351) = 17417 \cdot 304039 \cdot 7771457 \cdot 81400859 \cdot 97333853$.
BV, $262351, 81400859:$ 81400859 is inadmissible.

BW. $266837:$ $\Phi_7(266837) = 11173 \cdot 4856279 \cdot 8205037 \cdot 11347981 \cdot 71450513$.
BW, $266837, 4856279:$ 4856279 is inadmissible.

BX. $312979:$ $\Phi_7(312979) = 5419 \cdot 104987 \cdot 115123 \cdot 520241 \cdot 849703 \cdot 32464153$.
BX, $312979, 5419:$ $\Phi_3(5419) = 3 \cdot 31 \cdot 313 \cdot 1009$.
BX, $312979, 5419, 3:$ $3 \in X$.

BY. $339943:$ $\Phi_7(339943) = 310577 \cdot 677447 \cdot 5376673 \cdot 14529761 \cdot 93890399$.
BY, $339943, 677447:$ 677447 is inadmissible.

BZ. $402383:$ $\Phi_7(402383) = 8093 \cdot 61657 \cdot 85597 \cdot 878221 \cdot 5240887 \cdot 21591347$.
BZ, $402383, 5240887:$ $\Phi_3(5240887) = 3 \cdot 1153 \cdot 7867 \cdot 1009369$.
BZ, $402383, 5240887, 3:$ $3 \in X$.

CA. $502339:$ $\Phi_7(502339) = 5657 \cdot 42491 \cdot 46187 \cdot 4107881 \cdot 5256413 \cdot 67030433$.
CA, $502339, 42491:$ $\Phi_3(42491) = 19 \cdot 139 \cdot 683653$.
CA, $502339, 42491, 19:$ $19 \in X$.

CB. $594533:$ $\Phi_7(594533) = 10613 \cdot 27917 \cdot 61643 \cdot 1356083 \cdot 30972047 \cdot 57571781$.
CB, $594533, 1356083:$ 1356083 is inadmissible.

CC. $663823:$ $\Phi_7(663823) = 2129 \cdot 8681 \cdot 2538733 \cdot 5864489 \cdot 11462851 \cdot 27128711$.
CC, $663823, 27128711:$ 27128711 is inadmissible.

CD. $737981:$ $\Phi_7(737981) = 1303 \cdot 107927 \cdot 845531 \cdot 1981267 \cdot 10305569 \cdot 66535379$.
CD, $737981, 107927:$ 107927 is inadmissible.

CE. $746939:$ $\Phi_7(746939) = 2129 \cdot 2927 \cdot 3851 \cdot 39901 \cdot 127079 \cdot 31370683 \cdot 45494401$.
CE, $746939, 127079:$ 127079 is inadmissible.

CF. $755239:$ $\Phi_7(755239) = 5153 \cdot 11887 \cdot 583493 \cdot 1096859 \cdot 61078081 \cdot 77500193$.
CF, $755239, 61078081:$ $\Phi_2(61078081) = 2 \cdot 13 \cdot 113 \cdot 20789$.
CF, $755239, 61078081^*, 13:$ $13 \in X$.

CG. $820789:$ $\Phi_7(820789) = 1877 \cdot 10039 \cdot 3678823 \cdot 7103699 \cdot 7727273 \cdot 80355437$.
CG, $820789, 3678823:$ 3678823 is inadmissible.

CH. $1017209:$ $\Phi_7(1017209) = 7057 \cdot 127289 \cdot 376853 \cdot 1339297 \cdot 46078537 \cdot 53027731$.
CH, $1017209, 46078537:$ $\Phi_2(46078537) = 2 \cdot 11 \cdot 179 \cdot 11701$.
CH, $1017209, 46078537^*, 11:$ $11 \in X$.

CI. 1149521 : $\Phi_7(1149521) = 15373 \cdot 138181 \cdot 1419839 \cdot 1479059 \cdot 8363671 \cdot 61840549$.
CI, 1149521, 1479059 : 1479059 is inadmissible.

CJ. 1732909 : $\Phi_7(1732909) = 69259 \cdot 1282471 \cdot 1483021 \cdot 2094107 \cdot 2241709 \cdot 43793093$.
CJ, 1732909, 1282471 : 1282471 is inadmissible.

CK. 1742537 : $\Phi_7(1742537) = 1303 \cdot 7393 \cdot 22093 \cdot 36037 \cdot 5348449 \cdot 17815267 \cdot 38309251$.
CK, 1742537, 17815267 : 17815267 is inadmissible.

CL. 1951819 : $\Phi_7(1951819) = 97007 \cdot 330611 \cdot 630169 \cdot 10342907 \cdot 11740093 \cdot 22529207$.
CL, 1951819, 97007 : 97007 is inadmissible.

CM. 2110763 : $\Phi_7(2110763) = 13063 \cdot 378379 \cdot 2771693 \cdot 5800481 \cdot 25065013 \cdot 44400721$.
CM, 2110763, 378379 : 378379 is inadmissible.

CN. 2299163 : $\Phi_7(2299163) = 38011 \cdot 259169 \cdot 659569 \cdot 5430923 \cdot 42765997 \cdot 97879993$.
CN, 2299163, 5430923 : 5430923 is inadmissible.

CO. 3088091 : $\Phi_7(3088091) = 1499 \cdot 2801 \cdot 7547 \cdot 1135247 \cdot 24173101 \cdot 25799593 \cdot 38655919$.
CO, 3088091, 1135247 : $\Phi_3(1135247) = 13 \cdot 229 \cdot 271 \cdot 1249 \cdot 1279$.
CO, 3088091, 1135247, 13 : $13 \in X$.

CP. 4480403 : $\Phi_7(4480403) = 8821 \cdot 70099 \cdot 208993 \cdot 734021 \cdot 1252483 \cdot 1369607 \cdot 49712419$.
CP, 4480403, 70099 : 70099 is inadmissible.

CQ. 5418277 : $\Phi_7(5418277) = 197023 \cdot 657959 \cdot 6573071 \cdot 28477093 \cdot 29668493 \cdot 35147309$.
CQ, 5418277, 657959 : $\Phi_3(657959) = 13 \cdot 61 \cdot 271 \cdot 907 \cdot 2221$.
CQ, 5418277, 657959, 13 : $13 \in X$.

CR. 5654071 : $\Phi_7(5654071) = 2927 \cdot 7547 \cdot 59333 \cdot 5776457 \cdot 9909593 \cdot 14802173 \cdot 29419237$.
CR, 5654071, 5776457 : $\Phi_2(5776457) = 2 \cdot 3 \cdot 962743$.
CR, 5654071, 5776457*, 3 : $3 \in X$.

CS. 6414623 : $\Phi_7(6414623) = 3067 \cdot 59011 \cdot 190093 \cdot 246289 \cdot 2459801 \cdot 36326963 \cdot 92011039$.
CS, 6414623, 36326963 : 36326963 is inadmissible.

CT. 6450307 : $\Phi_7(6450307) = 7253 \cdot 32789 \cdot 33629 \cdot 46327 \cdot 251063 \cdot 331339 \cdot 901279 \cdot 2592829$.
CT, 6450307, 901279 : 901279 is inadmissible.

CU. 6499631 : $\Phi_7(6499631) = 4663 \cdot 22247 \cdot 482441 \cdot 751871 \cdot 5595059 \cdot 18452113 \cdot 19406941$.
CU, 6499631, 5595059 : 5595059 is inadmissible.

CV. 7163917 : $\Phi_7(7163917) = 13903 \cdot 232877 \cdot 460013 \cdot 490169 \cdot 2888747 \cdot 3732499 \cdot 17172877$.
CV, 7163917, 2888747 : 2888747 is inadmissible.

CW. 7532381 : $\Phi_7(7532381) = 24179 \cdot 3786119 \cdot 15453887 \cdot 49118371 \cdot 49365779 \cdot 53241889$.
CW, 7532381, 49118371 : 49118371 is inadmissible.

CX. 8104141 : $\Phi_7(8104141) = 19937 \cdot 34217 \cdot 729191 \cdot 1015561 \cdot 1699111 \cdot 5768869 \cdot 57211127$.
CX, 8104141, 57211127 : 57211127 is inadmissible.

CY. 9997441 : $\Phi_7(9997441) = 1373 \cdot 20707 \cdot 1089383 \cdot 2475019 \cdot 8841449 \cdot 15675983 \cdot 93978403$.
CY, 9997441, 1089383 : 1089383 is inadmissible.

CZ. 14257819 : $\Phi_7(14257819) = 69623 \cdot 148457 \cdot 244301 \cdot 1530019 \cdot 3633029 \cdot 7253107 \cdot 82518283$.
CZ, 14257819, 69623 : $\Phi_3(69623) = 19 \cdot 37 \cdot 43 \cdot 160357$.
CZ, 14257819, 69623, $19 \in X$.

DA. 16606193 : $\Phi_7(16606193) = 2591 \cdot 1339409 \cdot 1348747 \cdot 2516963 \cdot 2541701 \cdot 10134433 \cdot 69104869.$
DA, 16606193, 1348747 : 1348747 is inadmissible.

DB. 16955431 : $\Phi_7(16955431) = 3319 \cdot 815809 \cdot 1018907 \cdot 3669373 \cdot 10842119 \cdot 11932439 \cdot 18142097.$
DB, 16955431, 1018907 : 1018907 is inadmissible.

DC. 18786679 : $\Phi_7(18786679) = 6637 \cdot 852881 \cdot 1694393 \cdot 3354037 \cdot 5747239 \cdot 6603437 \cdot 36010451.$
DC, 18786679, 5747239 : 5747239 is inadmissible.

DD. 21101371 : $\Phi_7(21101371) = 1471 \cdot 25621 \cdot 29723 \cdot 38767 \cdot 126757 \cdot 15828457 \cdot 20260003 \cdot 50009261.$
DD, 21101371, 20260003 : 20260003 is inadmissible.

DE. 23150119 : $\Phi_7(23150119) = 50821 \cdot 638359 \cdot 1123403 \cdot 2347549 \cdot 4149881 \cdot 20643421 \cdot 21001177.$
DE, 23150119, 638359 : 638359 is inadmissible.

DF. 28234649 : $\Phi_7(28234649) = 1163 \cdot 8807 \cdot 161561 \cdot 500459 \cdot 1078757 \cdot 2027411 \cdot 6028457 \cdot 46399151.$
DF, 28234649, 2027411 : 2027411 is inadmissible.

DG. 28361629 : $\Phi_7(28361629) = 7477 \cdot 292531 \cdot 640949 \cdot 3785531 \cdot 29562317 \cdot 54942413 \cdot 60379747.$
DG, 28361629, 3785531 : 3785531 is inadmissible.

DH. 28578083 : $\Phi_7(28578083) = 1362551 \cdot 27985189 \cdot 46422433 \cdot 53190257 \cdot 66210901 \cdot 87383227.$
DH, 28578083, 1362551 : 1362551 is inadmissible.

DI. 30314399 : $\Phi_7(30314399) = 15443 \cdot 199697 \cdot 2437219 \cdot 2907577 \cdot 5152463 \cdot 69163459 \cdot 99649061.$
DI, 30314399, 2437219 : 2437219 is inadmissible.

DJ. 30823421 : $\Phi_7(30823421) = 2003 \cdot 6553 \cdot 25579 \cdot 249859 \cdot 2857709 \cdot 3456377 \cdot 14080837 \cdot 73505153.$
DJ, 30823421, 25579 : $\Phi_3(25579) = 3 \cdot 1129 \cdot 193183.$
DJ, 30823421, 25579, 3 : $3 \in X$.

DK. 33729187 : $\Phi_7(33729187) = 65003 \cdot 240899 \cdot 662369 \cdot 3182341 \cdot 21742757 \cdot 27410237 \cdot 74850161.$
DK, 33729187, 65003 : $\Phi_3(65003) = 13 \cdot 487 \cdot 667423.$
DK, 33729187, 65003, 13 : $13 \in X$.

DL. 39434663 : $\Phi_7(39434663) = 3557 \cdot 3613 \cdot 8387 \cdot 14281 \cdot 19181 \cdot 292223 \cdot 832721 \cdot 22835639 \cdot 22922047.$
DL, 39434663, 292223 : 292223 is inadmissible.

DM. 41981081 : $\Phi_7(41981081) = 3221 \cdot 19237 \cdot 140057 \cdot 503231 \cdot 2538803 \cdot 3414181 \cdot 8413021 \cdot 17189131.$
DM, 41981081, 2538803 : 2538803 is inadmissible.

DN. 44894383 : $\Phi_7(44894383) = 1933 \cdot 14869 \cdot 26209 \cdot 150893 \cdot 702913 \cdot 27240151 \cdot 41178523 \cdot 91355993.$
DN, 44894383, 27240151 : 27240151 is inadmissible.

DO. 50837341 : $\Phi_7(50837341) = 1163 \cdot 2843 \cdot 23899 \cdot 30059 \cdot 56393 \cdot 1055881 \cdot 3963317 \cdot 5527481 \cdot 5571343.$
DO, 50837341, 2843 : $\Phi_3(2843) = 13 \cdot 67 \cdot 9283.$
DO, 50837341, 2843, 13 : $13 \in X$.

DP. 61964431 : $\Phi_7(61964431) = 2521 \cdot 9829 \cdot 168869 \cdot 787879 \cdot 1712383 \cdot 2880739 \cdot 57176701 \cdot 60875039.$
DP, 61964431, 787879 : 787879 is inadmissible.

DQ. 68113807 : $\Phi_7(68113807) = 10613 \cdot 30829 \cdot 83791 \cdot 574813 \cdot 3701881 \cdot 10213001 \cdot 12842411 \cdot 13051697.$
DQ, 68113807, 12842411 : 12842411 is inadmissible.

DR. 74306809 : $\Phi_7(74306809) = 3851 \cdot 38921 \cdot 604031 \cdot 662719 \cdot 1496167 \cdot 3987229 \cdot 15379967 \cdot 30578689.$
DR, 74306809, 1496167 : 1496167 is inadmissible.

DS. 93058573 : $\Phi_7(93058573) = 10459 \cdot 16927 \cdot 399281 \cdot 512597 \cdot 1225183 \cdot 9568931 \cdot 35313391 \cdot 43292201.$

DS, $93058573, 1225183:$ 1225183 is inadmissible.

DT. $96397919:$ $\Phi_7(96397919) = 7001 \cdot 35533 \cdot 314693 \cdot 627901 \cdot 5457971 \cdot 5471887 \cdot 9429869 \cdot 57964453.$
DT, $96397919, 5471887:$ 5471887 is inadmissible.

A.5 Table of acceptable values

p	r	$\Phi_r(p)$
3	7	1093
3	11	$23 \cdot 3851$
3	13	797161
3	17	$1871 \cdot 34511$
3	19	$1597 \cdot 363889$
3	29	$59 \cdot 28537 \cdot 20381027$
3	31	$683 \cdot 102673 \cdot 4404047$
3	47	$1223 \cdot 21997 \cdot 5112661 \cdot 96656723$
5	7	19531
5	11	12207031
5	19	$191 \cdot 6271 \cdot 3981071$
7	7	$29 \cdot 4733$
7	11	$1123 \cdot 293459$
11	7	$43 \cdot 45319$
11	11	$15797 \cdot 1806113$
13	7	5229043
13	11	$23 \cdot 419 \cdot 859 \cdot 18041$
13	13	$53 \cdot 264031 \cdot 1803647$
17	7	25646167
19	7	$701 \cdot 70841$
19	11	$104281 \cdot 62060021$
23	7	$29 \cdot 5336717$
29	7	$7 \cdot 88009573$
31	13	$42407 \cdot 2426789 \cdot 7908811$
37	7	$71 \cdot 37140797$
43	7	$7 \cdot 5839 \cdot 158341$
47	13	$53 \cdot 2237 \cdot 14050609 \cdot 71265169$
59	7	$43 \cdot 281 \cdot 757 \cdot 4691$
67	7	$175897 \cdot 522061$
71	7	$7 \cdot 883 \cdot 21020917$
79	7	$281 \cdot 337 \cdot 1289 \cdot 2017$
83	13	$1249 \cdot 1396513 \cdot 1423319 \cdot 43580447$
97	7	$43 \cdot 967 \cdot 20241187$
109	7	$113 \cdot 281 \cdot 53306107$
113	7	$7 \cdot 44983 \cdot 6670903$
127	7	$7 \cdot 43 \cdot 86353 \cdot 162709$
131	7	$127 \cdot 189967 \cdot 211093$
139	7	$29 \cdot 2857 \cdot 87683177$
167	11	$23 \cdot 89 \cdot 331 \cdot 397 \cdot 1013 \cdot 32099 \cdot 1940599$
173	7	$3144079 \cdot 8576317$
191	7	$127 \cdot 197 \cdot 10627 \cdot 183569$
191	13	$131 \cdot 1483 \cdot 9049 \cdot 92041 \cdot 301627 \cdot 48552947$
197	7	$7 \cdot 29 \cdot 97847 \cdot 2957767$

p	r	$\Phi_r(p)$
199	7	$29 \cdot 211 \cdot 883 \cdot 11552213$
211	7	$7 \cdot 307189 \cdot 41233879$
223	7	$29 \cdot 491 \cdot 1709 \cdot 5076443$
239	7	$7 \cdot 29 \cdot 245561 \cdot 3754507$
269	7	$43 \cdot 211 \cdot 631 \cdot 2633 \cdot 25229$
271	7	$9170197 \cdot 43355341$
293	7	$43 \cdot 2197609 \cdot 6718489$
347	7	$39577763 \cdot 44236319$
359	7	$211 \cdot 449 \cdot 1303 \cdot 4019 \cdot 4327$
367	7	$113 \cdot 233437 \cdot 92882357$
389	7	$127 \cdot 337 \cdot 659 \cdot 827 \cdot 148933$
397	7	$29 \cdot 127 \cdot 927137 \cdot 1149457$
397	11	$11 \cdot 23 \cdot 67 \cdot 3323 \cdot 239273 \cdot 344587 \cdot 20993369$
401	7	$29 \cdot 337 \cdot 263047 \cdot 1621397$
409	7	$6133 \cdot 15919 \cdot 48063373$
431	7	$29 \cdot 953 \cdot 967 \cdot 1009 \cdot 238267$
439	7	$7883 \cdot 63841 \cdot 14255627$
509	7	$29 \cdot 2801 \cdot 10333 \cdot 20759803$
601	7	$631 \cdot 4832521 \cdot 15479857$
607	7	$54517 \cdot 415759 \cdot 2210419$
619	7	$3389 \cdot 3732919 \cdot 4453751$
653	7	$21757 \cdot 706763 \cdot 5049773$
691	11	$59951 \cdot 133717 \cdot 183041 \cdot 455489 \cdot 37187767$
769	7	$197 \cdot 12342821 \cdot 85161343$
853	7	$2647 \cdot 11824121 \cdot 12321989$
887	7	$5167 \cdot 6271651 \cdot 15045661$
919	7	$29 \cdot 43 \cdot 3851 \cdot 21407 \cdot 5866379$
953	7	$7 \cdot 29 \cdot 71 \cdot 113 \cdot 127 \cdot 379 \cdot 9566159$
977	7	$29 \cdot 5573 \cdot 914047 \cdot 5893273$
1063	7	$337 \cdot 2423 \cdot 1289513 \cdot 1371511$
1123	7	$113 \cdot 3823 \cdot 293147 \cdot 15852481$
1181	7	$71 \cdot 27791 \cdot 202021 \cdot 6812527$
1279	7	$56701 \cdot 3745631 \cdot 20627531$
1283	7	$29 \cdot 631 \cdot 3739 \cdot 24781 \cdot 2632673$
1301	7	$29 \cdot 43 \cdot 58735811 \cdot 66256471$
1303	7	$7 \cdot 67579 \cdot 190261 \cdot 54417721$
1451	7	$2381 \cdot 52584967 \cdot 74590391$
1453	7	$1051 \cdot 11117 \cdot 363119 \cdot 2219491$
1481	7	$953 \cdot 2087 \cdot 265007 \cdot 20033231$
1523	7	$71 \cdot 337 \cdot 449 \cdot 235537 \cdot 4935043$
1531	7	$29 \cdot 631 \cdot 9247939 \cdot 76148717$
1693	7	$43 \cdot 337 \cdot 7673 \cdot 37171 \cdot 5700731$
1823	7	$29 \cdot 71 \cdot 547 \cdot 5709019 \cdot 5711539$
1879	7	$29 \cdot 2017 \cdot 20664827 \cdot 36429751$
1949	7	$71 \cdot 113 \cdot 3137 \cdot 206263 \cdot 10563827$

p	r	$\Phi_r(p)$
2003	7	$7 \cdot 1289 \cdot 10627 \cdot 33601 \cdot 20053433$
2053	7	$29 \cdot 161869 \cdot 3482179 \cdot 4582817$
2141	7	$29 \cdot 1163 \cdot 11719 \cdot 112967 \cdot 2158157$
2381	7	$7 \cdot 43 \cdot 2689 \cdot 3613 \cdot 72997 \cdot 853903$
2383	7	$475637 \cdot 6770429 \cdot 56889841$
2473	11	$23 \cdot 463 \cdot 2927 \cdot 8647 \cdot 81071 \cdot 451793 \cdot 640531 \cdot 1353551$
2503	7	$757 \cdot 19013 \cdot 3591869 \cdot 4758517$
2633	7	$7 \cdot 29 \cdot 232919 \cdot 2103613 \cdot 3351223$
2657	7	$71 \cdot 631 \cdot 1289 \cdot 1991389 \cdot 3060667$
2713	7	$29^2 \cdot 43 \cdot 73361 \cdot 258469 \cdot 581729$
3041	7	$29 \cdot 337 \cdot 9871 \cdot 811651 \cdot 10103759$
3083	7	$71 \cdot 743 \cdot 4481 \cdot 62189 \cdot 58431409$
3089	7	$1303 \cdot 89237 \cdot 316793 \cdot 23592997$
3221	7	$7 \cdot 673 \cdot 10333 \cdot 248879 \cdot 92204351$
3301	7	$29^2 \cdot 911 \cdot 38669 \cdot 186733 \cdot 233941$
3313	7	$29 \cdot 211 \cdot 216259 \cdot 884857 \cdot 1129619$
3623	7	$43^3 \cdot 35911 \cdot 353263 \cdot 2242843$
3779	7	$197 \cdot 2311 \cdot 23773 \cdot 455407 \cdot 591053$
4013	7	$71 \cdot 281 \cdot 9829 \cdot 438047 \cdot 48632711$
4129	7	$5867 \cdot 17053 \cdot 714463 \cdot 69339047$
4133	7	$547 \cdot 54083 \cdot 5997223 \cdot 28099541$
4241	7	$29 \cdot 197 \cdot 137957 \cdot 463303 \cdot 15938189$
4243	7	$7 \cdot 421 \cdot 953 \cdot 8443 \cdot 9157 \cdot 26879273$
4283	7	$29 \cdot 29611 \cdot 41539 \cdot 100003 \cdot 1730891$
4289	7	$1471 \cdot 8807 \cdot 9619 \cdot 32467 \cdot 1538951$
4327	7	$7 \cdot 3221 \cdot 5503 \cdot 5657 \cdot 92401 \cdot 101221$
4423	7	$113 \cdot 180797 \cdot 5720863 \cdot 64072051$
4621	7	$7 \cdot 2532587 \cdot 22078253 \cdot 24881851$
4801	7	$29 \cdot 1583 \cdot 6959 \cdot 3578737 \cdot 10713347$
4957	7	$7 \cdot 127 \cdot 449 \cdot 8387 \cdot 68279 \cdot 64917119$
5399	7	$2731 \cdot 9941 \cdot 14811889 \cdot 61602479$
5689	7	$9479 \cdot 107941 \cdot 338773 \cdot 97821473$
5953	7	$4663 \cdot 1352107 \cdot 1591927 \cdot 4434949$
6067	7	$43 \cdot 71 \cdot 1373 \cdot 6581 \cdot 32803 \cdot 55120451$
6577	7	$29 \cdot 43 \cdot 71 \cdot 40993 \cdot 644029 \cdot 34633243$
6917	7	$7 \cdot 71 \cdot 187909 \cdot 13749289 \cdot 85307111$
7027	7	$491 \cdot 298327 \cdot 25883593 \cdot 31760177$
7759	7	$29^2 \cdot 71 \cdot 337 \cdot 1051 \cdot 2213 \cdot 2689 \cdot 1733929$
8009	7	$7 \cdot 43 \cdot 127 \cdot 491 \cdot 127247 \cdot 305873 \cdot 361313$
8053	7	$29 \cdot 71 \cdot 113 \cdot 379 \cdot 673 \cdot 67537 \cdot 68056451$
8243	7	$29 \cdot 883 \cdot 13259 \cdot 9842603 \cdot 93882587$
8681	7	$7 \cdot 62903 \cdot 285979 \cdot 1563101 \cdot 2174593$
9103	7	$631 \cdot 911 \cdot 514417 \cdot 1338331 \cdot 1437899$
9397	7	$617 \cdot 8779 \cdot 11579 \cdot 1551383 \cdot 7077197$
9403	7	$29 \cdot 2311 \cdot 2633 \cdot 9521 \cdot 54629 \cdot 7531651$

p	r	$\Phi_r(p)$
9463	7	$71 \cdot 6791 \cdot 15667 \cdot 4172099 \cdot 22786961$
9539	7	$673 \cdot 3504929 \cdot 6483611 \cdot 49266463$
9719	7	$281 \cdot 3067 \cdot 8219 \cdot 19937 \cdot 30773 \cdot 193957$
9787	7	$7 \cdot 127 \cdot 1385441 \cdot 7148849 \cdot 99819637$
9967	7	$29 \cdot 6427 \cdot 47293 \cdot 8157031 \cdot 13636253$
10889	7	$2003 \cdot 22093 \cdot 116341 \cdot 471997 \cdot 686057$
10939	7	$197 \cdot 1093 \cdot 205913 \cdot 2212673 \cdot 17466989$
10949	7	$7 \cdot 29 \cdot 197 \cdot 547 \cdot 1009 \cdot 6917 \cdot 25523 \cdot 442177$
11549	7	$29 \cdot 449 \cdot 236909 \cdot 21238211 \cdot 36220969$
11987	7	$211 \cdot 463 \cdot 673 \cdot 2689 \cdot 314161 \cdot 53416777$
12253	7	$43 \cdot 113 \cdot 379 \cdot 113723 \cdot 257993 \cdot 62639417$
12553	7	$29 \cdot 701 \cdot 1471 \cdot 49463 \cdot 594119 \cdot 4452841$
12637	7	$127 \cdot 701 \cdot 641747 \cdot 732299 \cdot 97347377$
12917	7	$953 \cdot 6007 \cdot 352661 \cdot 442961 \cdot 5194337$
13513	7	$43 \cdot 1303 \cdot 122081 \cdot 23736637 \cdot 37502711$
13723	7	$71 \cdot 239 \cdot 1492163 \cdot 4937941 \cdot 53420459$
15307	7	$29 \cdot 883 \cdot 2381 \cdot 51437 \cdot 1001953 \cdot 4093811$
15313	7	$71 \cdot 463 \cdot 547 \cdot 44633 \cdot 3918587 \cdot 4099943$
15601	7	$43 \cdot 2423 \cdot 3335263 \cdot 5823749 \cdot 7125049$
17011	7	$7 \cdot 6217 \cdot 311963 \cdot 18293101 \cdot 97574261$
17431	7	$7 \cdot 43 \cdot 127 \cdot 3347 \cdot 50527 \cdot 558881 \cdot 7764079$
17477	7	$547 \cdot 2549 \cdot 13931 \cdot 26619181 \cdot 55117819$
17491	7	$239 \cdot 827 \cdot 1093 \cdot 12097 \cdot 2287951 \cdot 4789219$
17609	7	$3109 \cdot 15289 \cdot 116747 \cdot 347873 \cdot 15444241$
18397	7	$7 \cdot 1933 \cdot 4957 \cdot 32411 \cdot 1343917 \cdot 13270643$
18859	7	$7 \cdot 5237 \cdot 6833 \cdot 39019 \cdot 184913 \cdot 24894269$
19441	7	$421 \cdot 2143 \cdot 4663 \cdot 15233 \cdot 383489 \cdot 2196979$
20441	7	$7 \cdot 29 \cdot 43 \cdot 659 \cdot 2213 \cdot 27763 \cdot 68279 \cdot 3023077$
21803	7	$29^2 \cdot 3221 \cdot 128857 \cdot 13311383 \cdot 23120623$
21961	7	$547 \cdot 171473 \cdot 178613 \cdot 640109 \cdot 10461221$
23339	7	$7 \cdot 29 \cdot 1051 \cdot 21841 \cdot 92779 \cdot 353137 \cdot 1058639$
23671	7	$29 \cdot 43 \cdot 239 \cdot 9829 \cdot 90203 \cdot 237301 \cdot 2805587$
24439	7	$337 \cdot 1723 \cdot 6287 \cdot 7043 \cdot 576689 \cdot 14370119$
24683	7	$7 \cdot 113 \cdot 2213 \cdot 2843 \cdot 18803 \cdot 67033 \cdot 36054103$
24809	7	$7 \cdot 43 \cdot 71 \cdot 1303 \cdot 362069 \cdot 1266371 \cdot 18262133$
25237	7	$29 \cdot 71 \cdot 9689 \cdot 641327 \cdot 1970417 \cdot 10248743$
25717	7	$29 \cdot 617 \cdot 1667 \cdot 305971 \cdot 332851 \cdot 95233517$
26833	7	$26041 \cdot 11780777 \cdot 19965779 \cdot 60941581$
27457	7	$29 \cdot 42463 \cdot 65171 \cdot 71261 \cdot 91813 \cdot 816047$
27763	7	$7 \cdot 16073 \cdot 7921691 \cdot 10907849 \cdot 47104457$
30557	7	$29 \cdot 449 \cdot 631 \cdot 10067 \cdot 90679 \cdot 99667 \cdot 1089047$
31667	7	$757 \cdot 7309 \cdot 1894873 \cdot 3503711 \cdot 27453203$
32051	7	$43 \cdot 71 \cdot 24151 \cdot 488797 \cdot 2163883 \cdot 13900769$
32653	7	$43 \cdot 113 \cdot 827 \cdot 831713 \cdot 6545477 \cdot 55409551$
33409	7	$43 \cdot 127 \cdot 10739 \cdot 303731 \cdot 2156183 \cdot 36206213$

p	r	$\Phi_r(p)$
35023	7	$29 \cdot 43 \cdot 71 \cdot 113 \cdot 1499 \cdot 4243 \cdot 330877 \cdot 87657277$
35983	7	$29 \cdot 43 \cdot 113 \cdot 21001 \cdot 30983 \cdot 2938307 \cdot 8057323$
39607	7	$7 \cdot 43 \cdot 421 \cdot 2549 \cdot 28477 \cdot 18879253 \cdot 22230293$
41203	7	$7 \cdot 29 \cdot 211 \cdot 1163 \cdot 6329 \cdot 16339 \cdot 340397 \cdot 2790481$
41759	7	$617 \cdot 48371 \cdot 84463 \cdot 25517689 \cdot 82439029$
41941	7	$29 \cdot 43 \cdot 4649 \cdot 1506457 \cdot 13157341 \cdot 47368777$
42403	7	$113 \cdot 239 \cdot 3851 \cdot 140869 \cdot 8153447 \cdot 48661243$
42577	7	$71 \cdot 127 \cdot 827 \cdot 2297 \cdot 13567 \cdot 2320361 \cdot 11048227$
43721	7	$911 \cdot 71317 \cdot 1158823 \cdot 6418259 \cdot 14454553$
45137	7	$7 \cdot 6091 \cdot 47951 \cdot 156269 \cdot 428807 \cdot 61728647$
45263	7	$7 \cdot 29 \cdot 225499 \cdot 1291991 \cdot 3314893 \cdot 43863163$
45341	7	$113 \cdot 127 \cdot 13259 \cdot 2515003 \cdot 2804117 \cdot 6474833$
46327	7	$7 \cdot 43 \cdot 127373 \cdot 1260673 \cdot 7805981 \cdot 26202373$
48131	7	$29 \cdot 113 \cdot 213613 \cdot 235747 \cdot 3276491 \cdot 22993181$
48311	7	$6751571 \cdot 7550173 \cdot 8327677 \cdot 29950187$
48473	7	$113 \cdot 30829 \cdot 42743 \cdot 241249 \cdot 352073 \cdot 1025669$
48809	7	$43 \cdot 71 \cdot 113 \cdot 4831 \cdot 97609 \cdot 6382097 \cdot 13023053$
50261	7	$7 \cdot 5923 \cdot 88747 \cdot 168631 \cdot 2245489 \cdot 11570539$
52631	7	$29 \cdot 71 \cdot 38711 \cdot 459383 \cdot 7327601 \cdot 79219211$
53831	7	$7 \cdot 29 \cdot 39341 \cdot 104651 \cdot 257489 \cdot 269221 \cdot 420001$
55787	7	$29 \cdot 43 \cdot 197 \cdot 239 \cdot 2292053 \cdot 10808939 \cdot 20723711$
56053	7	$29 \cdot 743 \cdot 104707 \cdot 1805833 \cdot 2248681 \cdot 3385579$
56431	7	$11047 \cdot 187573 \cdot 597367 \cdot 1912541 \cdot 13641041$
58453	7	$43 \cdot 71 \cdot 2143 \cdot 4019 \cdot 4229 \cdot 5264099 \cdot 68142733$
59467	7	$43 \cdot 1373 \cdot 140449 \cdot 469631 \cdot 2847139 \cdot 3988783$
61909	7	$7 \cdot 29 \cdot 4663 \cdot 34693 \cdot 130439 \cdot 143333 \cdot 91700729$
63587	7	$13063 \cdot 380059 \cdot 1341257 \cdot 1632751 \cdot 6079823$
63659	7	$7 \cdot 127 \cdot 211 \cdot 827 \cdot 414331 \cdot 18362807 \cdot 56388361$
64661	7	$29 \cdot 13931 \cdot 67061 \cdot 233549 \cdot 572909 \cdot 20162633$
64853	7	$71 \cdot 701 \cdot 12433 \cdot 800143 \cdot 6421157 \cdot 23402051$
66529	7	$7 \cdot 9521 \cdot 36107 \cdot 50821 \cdot 20364457 \cdot 34816447$
67021	7	$337 \cdot 21841 \cdot 28183 \cdot 183709 \cdot 209189 \cdot 11368757$
67477	7	$29 \cdot 113 \cdot 827 \cdot 5237 \cdot 27077 \cdot 4379033 \cdot 56091589$
70223	7	$43 \cdot 211 \cdot 1429 \cdot 3978899 \cdot 35870297 \cdot 64803943$
70913	7	$743 \cdot 5923 \cdot 13693 \cdot 119533 \cdot 365513 \cdot 48299371$
71209	7	$46439 \cdot 128969 \cdot 160651 \cdot 10145059 \cdot 13357009$
73589	7	$29 \cdot 43 \cdot 967 \cdot 7757 \cdot 52529 \cdot 13546121 \cdot 23860663$
73819	7	$113 \cdot 127 \cdot 3347 \cdot 771653 \cdot 65887067 \cdot 66260503$
74177	7	$29 \cdot 113 \cdot 7043 \cdot 9045947 \cdot 9943207 \cdot 80242933$
75367	7	$29 \cdot 113 \cdot 281 \cdot 102593 \cdot 159503 \cdot 275059 \cdot 44217881$
75557	7	$281 \cdot 701 \cdot 55217 \cdot 168533 \cdot 5049521 \cdot 20101187$
76753	7	$43 \cdot 379 \cdot 1093 \cdot 4995299 \cdot 26776331 \cdot 85809907$
78653	7	$7 \cdot 2549 \cdot 109481 \cdot 1536221 \cdot 4705681 \cdot 16765421$
80677	7	$127 \cdot 197 \cdot 491 \cdot 743 \cdot 1051 \cdot 24809 \cdot 66403 \cdot 17448733$
83071	7	$113 \cdot 211 \cdot 1093 \cdot 1163 \cdot 7393 \cdot 23920163 \cdot 61313519$

p	r	$\Phi_r(p)$
85597	7	$7 \cdot 617 \cdot 39509 \cdot 8658679 \cdot 8851837 \cdot 30074059$
89083	7	$7 \cdot 29^2 \cdot 534199 \cdot 1846909 \cdot 7210043 \cdot 11934203$
90149	7	$43 \cdot 2143 \cdot 71429 \cdot 1223587 \cdot 2792189 \cdot 23868517$
91153	7	$281^2 \cdot 162359 \cdot 1860517 \cdot 2205449 \cdot 10904629$
92233	7	$7 \cdot 43 \cdot 1289 \cdot 1597 \cdot 6833 \cdot 25579 \cdot 513367 \cdot 11073259$
92503	7	$239 \cdot 247031 \cdot 8538797 \cdot 22452151 \cdot 55352291$
93251	7	$29 \cdot 2129 \cdot 2437 \cdot 25439 \cdot 148471 \cdot 385939 \cdot 2998031$
94253	7	$421 \cdot 1706363 \cdot 1886347 \cdot 22400533 \cdot 23096291$
96907	7	$211 \cdot 774691 \cdot 3782773 \cdot 31910201 \cdot 41974409$
99551	7	$29 \cdot 547 \cdot 673 \cdot 8387 \cdot 783931 \cdot 1180901 \cdot 11743019$
99623	7	$43 \cdot 119869 \cdot 48536153 \cdot 49730507 \cdot 78577549$
103981	7	$29 \cdot 71 \cdot 2129 \cdot 2339 \cdot 476029 \cdot 4352279 \cdot 59499833$
108949	7	$7 \cdot 29 \cdot 2549 \cdot 8779 \cdot 415031 \cdot 23933771 \cdot 37063027$
109793	7	$2731 \cdot 6287 \cdot 46831 \cdot 314581 \cdot 950111 \cdot 7288639$
113287	7	$2339 \cdot 3319 \cdot 5419 \cdot 2085931 \cdot 4027927 \cdot 5980619$
115337	7	$43 \cdot 659 \cdot 49043 \cdot 3420691 \cdot 5744957 \cdot 86195411$
118189	7	$7 \cdot 71 \cdot 57373 \cdot 60397 \cdot 230567 \cdot 1121051 \cdot 6122999$
120349	7	$43 \cdot 547 \cdot 358541 \cdot 1970291 \cdot 10184959 \cdot 17954539$
120569	7	$7 \cdot 29 \cdot 554233 \cdot 7019153 \cdot 58047053 \cdot 67013801$
120779	7	$7 \cdot 29 \cdot 43 \cdot 127 \cdot 11299 \cdot 3491881 \cdot 5076821 \cdot 13979533$
123493	7	$211 \cdot 449 \cdot 827 \cdot 57667 \cdot 143263 \cdot 880909 \cdot 6220579$
124601	7	$7 \cdot 757 \cdot 953 \cdot 5279 \cdot 455687 \cdot 10627373 \cdot 28986889$
124633	7	$29 \cdot 2801 \cdot 300931 \cdot 394577 \cdot 17641961 \cdot 22026481$
126551	7	$29 \cdot 2511167 \cdot 29664223 \cdot 38610461 \cdot 49247633$
128833	7	$211 \cdot 701 \cdot 1723 \cdot 160441 \cdot 161267 \cdot 205423 \cdot 3375751$
137699	7	$29 \cdot 71 \cdot 591319 \cdot 5263679 \cdot 24672733 \cdot 43112483$
140009	7	$35869 \cdot 545651 \cdot 1282681 \cdot 9028013 \cdot 33234853$
143729	7	$113 \cdot 9199 \cdot 89657 \cdot 1985047 \cdot 4602109 \cdot 10354723$
144407	7	$29 \cdot 113 \cdot 127 \cdot 2017 \cdot 9654247 \cdot 14866013 \cdot 75272009$
144577	7	$43 \cdot 491 \cdot 300301 \cdot 5468891 \cdot 14520353 \cdot 18139073$
146801	7	$197 \cdot 911 \cdot 2801 \cdot 49547 \cdot 343897 \cdot 403957 \cdot 2892667$
148549	7	$281 \cdot 1429 \cdot 1709 \cdot 44381 \cdot 62539 \cdot 91967 \cdot 61342387$
153107	7	$29 \cdot 3109 \cdot 12097 \cdot 16489523 \cdot 17273131 \cdot 41466517$
154183	7	$7 \cdot 449 \cdot 2017 \cdot 53719 \cdot 71387 \cdot 9396731 \cdot 58809521$
155833	7	$29 \cdot 127 \cdot 719027 \cdot 12534761 \cdot 14455757 \cdot 29843899$
156703	7	$7 \cdot 29 \cdot 43 \cdot 827 \cdot 5209 \cdot 4776647 \cdot 8992691 \cdot 9167047$
156887	7	$71 \cdot 6679 \cdot 27539 \cdot 1839923 \cdot 18567557 \cdot 33423517$
162209	7	$71 \cdot 617 \cdot 112687 \cdot 665141 \cdot 67552997 \cdot 82125247$
162973	7	$127 \cdot 1303 \cdot 47657 \cdot 3932993 \cdot 8453509 \cdot 71459767$
166207	7	$827 \cdot 4019 \cdot 11719 \cdot 3595733 \cdot 4190999 \cdot 35915293$
172489	7	$43 \cdot 71 \cdot 197 \cdot 1723 \cdot 8233 \cdot 231547 \cdot 2129107 \cdot 6261781$
174289	7	$631 \cdot 1307923 \cdot 20800529 \cdot 29855141 \cdot 54691183$
174637	7	$7 \cdot 71 \cdot 491 \cdot 3571 \cdot 33587 \cdot 315281 \cdot 832987 \cdot 3690499$
174749	7	$7 \cdot 29 \cdot 1933 \cdot 208699 \cdot 1906871 \cdot 2171261 \cdot 83986421$
181739	7	$29 \cdot 43^2 \cdot 127 \cdot 757 \cdot 4691 \cdot 44507 \cdot 3790739 \cdot 8831593$

p	r	$\Phi_r(p)$
187049	7	$351779 \cdot 511463 \cdot 1506653 \cdot 5043011 \cdot 31329061$
191693	7	$7561 \cdot 11887 \cdot 14869 \cdot 16759 \cdot 89839 \cdot 118399 \cdot 208279$
191801	7	$7 \cdot 29 \cdot 43 \cdot 71 \cdot 11789 \cdot 1607327 \cdot 44459479 \cdot 95354029$
196657	7	$449 \cdot 617 \cdot 140729 \cdot 1982191 \cdot 20355287 \cdot 36772303$
204599	7	$71 \cdot 127 \cdot 883 \cdot 5153 \cdot 5419 \cdot 81439 \cdot 150907 \cdot 26845981$
218971	7	$197 \cdot 1471 \cdot 2647 \cdot 9871 \cdot 1557613 \cdot 2178877 \cdot 4289783$
219031	7	$7 \cdot 29 \cdot 197 \cdot 22541 \cdot 174721 \cdot 183317 \cdot 771653 \cdot 4955987$
220217	7	$29 \cdot 211 \cdot 239 \cdot 2423 \cdot 4201 \cdot 41231 \cdot 3201619 \cdot 58040221$
236641	7	$449 \cdot 23899 \cdot 154981 \cdot 735953 \cdot 6614273 \cdot 21692273$
242617	7	$43 \cdot 7253 \cdot 140603 \cdot 3349501 \cdot 16050539 \cdot 86512469$
244457	7	$29 \cdot 883 \cdot 1583 \cdot 3361 \cdot 6203 \cdot 32341 \cdot 2406307 \cdot 3244907$
246319	7	$71 \cdot 1303 \cdot 1709 \cdot 6217 \cdot 6553 \cdot 87683 \cdot 141121 \cdot 2802311$
254377	7	$953 \cdot 2269 \cdot 882967 \cdot 1007609 \cdot 3497089 \cdot 40271533$
262351	7	$17417 \cdot 304039 \cdot 7771457 \cdot 81400859 \cdot 97333853$
264139	7	$7 \cdot 29 \cdot 239 \cdot 33811 \cdot 49547 \cdot 82601 \cdot 620159 \cdot 81571631$
266837	7	$11173 \cdot 4856279 \cdot 8205037 \cdot 11347981 \cdot 71450513$
267139	7	$29 \cdot 71 \cdot 7603 \cdot 57793 \cdot 814493 \cdot 17651341 \cdot 27941117$
275719	7	$29 \cdot 127 \cdot 421 \cdot 12923 \cdot 15359 \cdot 25733 \cdot 2735377 \cdot 20280751$
285559	7	$7 \cdot 29 \cdot 449 \cdot 1667 \cdot 3389 \cdot 14771 \cdot 210071 \cdot 224267 \cdot 1513163$
288109	7	$29 \cdot 6343 \cdot 6553 \cdot 17207 \cdot 75181 \cdot 6354013 \cdot 57722911$
290531	7	$463 \cdot 967 \cdot 812491 \cdot 819253 \cdot 38938117 \cdot 51824767$
304501	7	$7 \cdot 883 \cdot 69427 \cdot 430739 \cdot 532099 \cdot 882967 \cdot 9179003$
311569	7	$113 \cdot 33391 \cdot 1011599 \cdot 2509963 \cdot 4149811 \cdot 23009911$
312551	7	$7 \cdot 113 \cdot 127 \cdot 281 \cdot 119183 \cdot 1936747 \cdot 3272473 \cdot 43719677$
312979	7	$5419 \cdot 104987 \cdot 115123 \cdot 520241 \cdot 849703 \cdot 32464153$
313289	7	$71 \cdot 883 \cdot 710627 \cdot 6754567 \cdot 55483079 \cdot 56630897$
317729	7	$239 \cdot 25873 \cdot 60383 \cdot 628391 \cdot 57408821 \cdot 76379101$
327443	7	$43 \cdot 1429 \cdot 9437 \cdot 112603 \cdot 156913 \cdot 8109557 \cdot 14834569$
336373	7	$239 \cdot 673 \cdot 3613 \cdot 8821 \cdot 120919 \cdot 38839123 \cdot 60167969$
339943	7	$310577 \cdot 677447 \cdot 5376673 \cdot 14529761 \cdot 93890399$
344759	7	$29 \cdot 113 \cdot 337 \cdot 9157 \cdot 25537 \cdot 186761 \cdot 776077 \cdot 44861573$
345311	7	$7 \cdot 43 \cdot 491 \cdot 617 \cdot 2731 \cdot 5279 \cdot 53173 \cdot 1412461 \cdot 17170763$
354551	7	$7 \cdot 43 \cdot 71 \cdot 6581 \cdot 206501 \cdot 386989 \cdot 4076003 \cdot 43361221$
358223	7	$239 \cdot 8933 \cdot 28057 \cdot 13193419 \cdot 34867841 \cdot 76683713$
392087	7	$29 \cdot 1303 \cdot 11159 \cdot 74201 \cdot 895007 \cdot 4153759 \cdot 31235653$
400837	7	$211 \cdot 127583 \cdot 172243 \cdot 1970921 \cdot 5289901 \cdot 85797433$
401593	7	$43 \cdot 1289 \cdot 14827 \cdot 51871 \cdot 153371 \cdot 20750843 \cdot 30920009$
402383	7	$8093 \cdot 61657 \cdot 85597 \cdot 878221 \cdot 5240887 \cdot 21591347$
404267	7	$29 \cdot 631 \cdot 31627 \cdot 156913 \cdot 375103 \cdot 2655577 \cdot 48256643$
406631	7	$7 \cdot 39971 \cdot 926983 \cdot 12504647 \cdot 18117401 \cdot 76934621$
413477	7	$7 \cdot 29 \cdot 1303 \cdot 3147649 \cdot 12927349 \cdot 15906563 \cdot 29187481$
432923	7	$7 \cdot 197 \cdot 1373 \cdot 1429 \cdot 6203 \cdot 7211 \cdot 57709 \cdot 76231 \cdot 12365893$
434029	7	$7 \cdot 113 \cdot 281 \cdot 4523 \cdot 16927 \cdot 2966083 \cdot 8201663 \cdot 16148749$
441841	7	$7 \cdot 43 \cdot 127 \cdot 281 \cdot 449 \cdot 9059 \cdot 2587943 \cdot 7274807 \cdot 9045191$
445229	7	$7 \cdot 337 \cdot 463 \cdot 1933 \cdot 11579 \cdot 210869 \cdot 32508197 \cdot 46481933$

p	r	$\Phi_r(p)$
449333	7	$29 \cdot 71 \cdot 1709 \cdot 3137 \cdot 4999 \cdot 8681 \cdot 12713 \cdot 26993 \cdot 50066339$
460181	7	$7 \cdot 71 \cdot 181609 \cdot 222461 \cdot 3397549 \cdot 9759121 \cdot 14264251$
463987	7	$29 \cdot 224309 \cdot 384889 \cdot 4913497 \cdot 9379861 \cdot 86470609$
469237	7	$43 \cdot 491 \cdot 1051 \cdot 7477 \cdot 9227 \cdot 205703 \cdot 1287217 \cdot 26334281$
472457	7	$43 \cdot 1460957 \cdot 2076803 \cdot 2178653 \cdot 4680733 \cdot 8359331$
472721	7	$281 \cdot 18523 \cdot 41959 \cdot 9156701 \cdot 69643771 \cdot 80124661$
483503	7	$43 \cdot 6833 \cdot 42071 \cdot 93941 \cdot 235607 \cdot 640669 \cdot 72888019$
493397	7	$29^2 \cdot 127 \cdot 1163 \cdot 2129 \cdot 4229 \cdot 26041 \cdot 50177 \cdot 71359 \cdot 138349$
494677	7	$7 \cdot 29 \cdot 71 \cdot 491 \cdot 1597 \cdot 14533 \cdot 595351 \cdot 6774503 \cdot 22119973$
502339	7	$5657 \cdot 42491 \cdot 46187 \cdot 4107881 \cdot 5256413 \cdot 67030433$
524389	7	$43 \cdot 3739 \cdot 4397 \cdot 203897 \cdot 210533 \cdot 9011059 \cdot 76038761$
529421	7	$71 \cdot 113 \cdot 911 \cdot 16087 \cdot 45403 \cdot 136319 \cdot 1756567 \cdot 17225503$
534883	7	$29 \cdot 337 \cdot 463 \cdot 2900437 \cdot 4130309 \cdot 12971197 \cdot 33305707$
560093	7	$29 \cdot 71 \cdot 109313 \cdot 817279 \cdot 1927507 \cdot 6269971 \cdot 13886783$
563503	7	$127 \cdot 1933 \cdot 2491847 \cdot 14002283 \cdot 50976143 \cdot 73325561$
575557	7	$29 \cdot 63463 \cdot 102607 \cdot 121633 \cdot 188189 \cdot 336827 \cdot 24967937$
578131	7	$7 \cdot 29 \cdot 2633 \cdot 159223 \cdot 177269 \cdot 188147 \cdot 539323 \cdot 24390829$
578407	7	$281 \cdot 757 \cdot 51871 \cdot 80473 \cdot 894181 \cdot 953093 \cdot 49483939$
592919	7	$43 \cdot 6833 \cdot 60607 \cdot 237707 \cdot 704243 \cdot 2362571 \cdot 6169087$
593083	7	$7 \cdot 71^2 \cdot 827 \cdot 4481 \cdot 6917 \cdot 83791 \cdot 15279601 \cdot 37581251$
594533	7	$10613 \cdot 27917 \cdot 61643 \cdot 1356083 \cdot 30972047 \cdot 57571781$
595877	7	$71 \cdot 113 \cdot 66809 \cdot 136403 \cdot 2231881 \cdot 4590251 \cdot 59763677$
654343	7	$29 \cdot 1093 \cdot 58451 \cdot 79423 \cdot 878641 \cdot 21934291 \cdot 27678463$
655351	7	$281 \cdot 808039 \cdot 1383509 \cdot 3056789 \cdot 4971583 \cdot 16594481$
663823	7	$2129 \cdot 8681 \cdot 2538733 \cdot 5864489 \cdot 11462851 \cdot 27128711$
677021	7	$29 \cdot 690887 \cdot 743989 \cdot 7240073 \cdot 23207297 \cdot 38448061$
692353	7	$29 \cdot 71 \cdot 449 \cdot 911 \cdot 1499 \cdot 128969 \cdot 204331 \cdot 1443989 \cdot 2292767$
704731	7	$43 \cdot 127 \cdot 337 \cdot 150011 \cdot 304739 \cdot 323009 \cdot 626011 \cdot 7200971$
732157	7	$29 \cdot 113 \cdot 127 \cdot 239 \cdot 2689 \cdot 170213 \cdot 341993 \cdot 400331 \cdot 24713137$
737129	7	$7 \cdot 29 \cdot 113 \cdot 491 \cdot 883 \cdot 911 \cdot 16073 \cdot 109201 \cdot 2807239 \cdot 3593549$
737981	7	$1303 \cdot 107927 \cdot 845531 \cdot 1981267 \cdot 10305569 \cdot 66535379$
741809	7	$43 \cdot 127 \cdot 197 \cdot 211 \cdot 1163 \cdot 13903 \cdot 200383 \cdot 12055723 \cdot 18792733$
746939	7	$2129 \cdot 2927 \cdot 3851 \cdot 39901 \cdot 127079 \cdot 31370683 \cdot 45494401$
755239	7	$5153 \cdot 11887 \cdot 583493 \cdot 1096859 \cdot 61078081 \cdot 77500193$
757271	7	$29 \cdot 43 \cdot 16381 \cdot 197009 \cdot 16811131 \cdot 36164437 \cdot 77079157$
784457	7	$29^2 \cdot 743 \cdot 1429 \cdot 18061 \cdot 21997487 \cdot 25506643 \cdot 25753141$
820789	7	$1877 \cdot 10039 \cdot 3678823 \cdot 7103699 \cdot 7727273 \cdot 80355437$
822989	7	$827 \cdot 77491 \cdot 1248353 \cdot 5998217 \cdot 14279539 \cdot 45345497$
844957	7	$7 \cdot 37493 \cdot 150431 \cdot 662551 \cdot 889687 \cdot 1603267 \cdot 9753493$
851297	7	$127 \cdot 211 \cdot 4271 \cdot 10151 \cdot 52963 \cdot 804329 \cdot 1531181 \cdot 5022613$
882461	7	$29 \cdot 449 \cdot 186271 \cdot 231631 \cdot 7643609 \cdot 9319619 \cdot 11800237$
899291	7	$7 \cdot 127 \cdot 337 \cdot 3319 \cdot 8821 \cdot 11159 \cdot 282703 \cdot 330569 \cdot 57826567$
921259	7	$29 \cdot 178151 \cdot 9267917 \cdot 16071007 \cdot 27967367 \cdot 28407163$
923599	7	$29 \cdot 757 \cdot 3823 \cdot 16493 \cdot 223217 \cdot 422689 \cdot 1165711 \cdot 4077221$
930481	7	$29 \cdot 43 \cdot 1429 \cdot 4789 \cdot 14281 \cdot 14827 \cdot 35099 \cdot 542123 \cdot 18875459$

p	r	$\Phi_r(p)$
971039	7	$113 \cdot 491 \cdot 71527 \cdot 2404823 \cdot 2447551 \cdot 4459687 \cdot 8047691$
982339	7	$7 \cdot 43 \cdot 113 \cdot 1597 \cdot 4733 \cdot 6637 \cdot 5035339 \cdot 8371063 \cdot 12493993$
982603	7	$29 \cdot 127 \cdot 197 \cdot 883 \cdot 1933 \cdot 2437 \cdot 44059 \cdot 78041321 \cdot 86734859$
998633	7	$127 \cdot 70841 \cdot 1564907 \cdot 15453299 \cdot 56513297 \cdot 80665369$
1006933	7	$29 \cdot 9283 \cdot 65843 \cdot 93703 \cdot 299419 \cdot 22153909 \cdot 94607311$
1017209	7	$7057 \cdot 127289 \cdot 376853 \cdot 1339297 \cdot 46078537 \cdot 53027731$
1019663	7	$7 \cdot 29 \cdot 43 \cdot 71 \cdot 8513 \cdot 288527 \cdot 1269311 \cdot 5819773 \cdot 99948059$
1027931	7	$43 \cdot 2927 \cdot 24876671 \cdot 59459527 \cdot 64857563 \cdot 97705427$
1028329	7	$7 \cdot 421 \cdot 49057 \cdot 630827 \cdot 22137277 \cdot 22320607 \cdot 26240453$
1054373	7	$29 \cdot 547 \cdot 883 \cdot 3221 \cdot 16871 \cdot 263201 \cdot 74239831 \cdot 92377027$
1132721	7	$127 \cdot 239 \cdot 337 \cdot 1289 \cdot 246289 \cdot 842003 \cdot 12481771 \cdot 61889759$
1135451	7	$239 \cdot 827 \cdot 4663 \cdot 42967 \cdot 89041 \cdot 223007 \cdot 309583 \cdot 8802809$
1149521	7	$15373 \cdot 138181 \cdot 1419839 \cdot 1479059 \cdot 8363671 \cdot 61840549$
1149803	7	$113 \cdot 547 \cdot 2143 \cdot 4691 \cdot 29723 \cdot 46187 \cdot 39756991 \cdot 68133661$
1233187	7	$29 \cdot 197 \cdot 373003 \cdot 887153 \cdot 3892631 \cdot 5162011 \cdot 92584171$
1236757	7	$29 \cdot 135017 \cdot 176849 \cdot 356077 \cdot 379849 \cdot 882491 \cdot 43296457$
1242781	7	$7 \cdot 547 \cdot 13903 \cdot 48413 \cdot 265511 \cdot 1095403 \cdot 2171261 \cdot 2263829$
1246631	7	$7 \cdot 4397 \cdot 458963 \cdot 647809 \cdot 1381997 \cdot 16704157 \cdot 17767121$
1249099	7	$43 \cdot 47251 \cdot 119687 \cdot 277747 \cdot 524189 \cdot 1847539 \cdot 58066303$
1286953	7	$29 \cdot 113 \cdot 25999 \cdot 6940823 \cdot 14275031 \cdot 16237369 \cdot 33146653$
1310251	7	$43 \cdot 113 \cdot 3347 \cdot 25733 \cdot 855191 \cdot 1249669 \cdot 1599809 \cdot 7071443$
1323967	7	$7 \cdot 71 \cdot 12097 \cdot 17389 \cdot 446041 \cdot 975157 \cdot 1999957 \cdot 59222213$
1328923	7	$7 \cdot 421 \cdot 107269 \cdot 229139 \cdot 10740689 \cdot 80014481 \cdot 88480001$
1385441	7	$7 \cdot 29 \cdot 13693 \cdot 342203 \cdot 353081 \cdot 905213 \cdot 934613 \cdot 24888179$
1416931	7	$29 \cdot 1429 \cdot 39971 \cdot 75181 \cdot 24696421 \cdot 30183203 \cdot 87178589$
1417831	7	$43 \cdot 71 \cdot 302597 \cdot 5249539 \cdot 7665491 \cdot 8279489 \cdot 26393137$
1458727	7	$71 \cdot 5531 \cdot 31991 \cdot 135787 \cdot 6083141 \cdot 30098699 \cdot 30847559$
1484701	7	$7 \cdot 71 \cdot 39313 \cdot 5420017 \cdot 26778473 \cdot 45707803 \cdot 82635169$
1485019	7	$29 \cdot 127 \cdot 421^2 \cdot 449 \cdot 283669 \cdot 769231 \cdot 2228927 \cdot 75234251$
1486847	7	$113 \cdot 27749 \cdot 46957 \cdot 180503 \cdot 419651 \cdot 22780171 \cdot 42524791$
1545217	7	$127 \cdot 1499 \cdot 78583 \cdot 1595273 \cdot 4021249 \cdot 4059581 \cdot 34940249$
1630021	7	$7 \cdot 113 \cdot 13679 \cdot 477863 \cdot 513899 \cdot 984859 \cdot 1458619 \cdot 4913959$
1652921	7	$127 \cdot 281 \cdot 421 \cdot 110083 \cdot 140869 \cdot 1516369 \cdot 3468851 \cdot 16641577$
1660063	7	$29 \cdot 32117 \cdot 101081 \cdot 582541 \cdot 2594677 \cdot 6035737 \cdot 24367169$
1709483	7	$29^2 \cdot 197 \cdot 953 \cdot 24809 \cdot 305621 \cdot 358667 \cdot 6433841 \cdot 9033991$
1732909	7	$69259 \cdot 1282471 \cdot 1483021 \cdot 2094107 \cdot 2241709 \cdot 43793093$
1740199	7	$29 \cdot 449 \cdot 6917 \cdot 7001 \cdot 11383 \cdot 39901 \cdot 61223 \cdot 337751 \cdot 4689427$
1740589	7	$43 \cdot 911 \cdot 2969 \cdot 4663 \cdot 23059 \cdot 5441311 \cdot 7725271 \cdot 52900219$
1742537	7	$1303 \cdot 7393 \cdot 22093 \cdot 36037 \cdot 5348449 \cdot 17815267 \cdot 38309251$
1793101	7	$673 \cdot 953 \cdot 147211 \cdot 231967 \cdot 6094229 \cdot 7934921 \cdot 31382891$
1795247	7	$113 \cdot 8779 \cdot 305761 \cdot 418181 \cdot 683957 \cdot 18912461 \cdot 20403223$
1827311	7	$281 \cdot 967 \cdot 869233 \cdot 1005551 \cdot 1834981 \cdot 2774129 \cdot 30792413$
1899047	7	$127 \cdot 197 \cdot 337 \cdot 164837 \cdot 908573 \cdot 1938161 \cdot 2651111 \cdot 7229069$
1909489	7	$7 \cdot 617 \cdot 5531 \cdot 42043 \cdot 91939 \cdot 4324979 \cdot 4671101 \cdot 25984813$
1918537	7	$743 \cdot 28813 \cdot 137453 \cdot 340201 \cdot 359549 \cdot 1970543 \cdot 70308883$

p	r	$\Phi_r(p)$
1939331	7	$71 \cdot 2689 \cdot 4523 \cdot 17683 \cdot 33181 \cdot 219647 \cdot 12133031 \cdot 39399991$
1950853	7	$29^2 \cdot 1877 \cdot 3823 \cdot 7127 \cdot 42169 \cdot 572881 \cdot 3453563 \cdot 15362117$
1951819	7	$97007 \cdot 330611 \cdot 630169 \cdot 10342907 \cdot 11740093 \cdot 22529207$
1992937	7	$43 \cdot 10753 \cdot 101627 \cdot 678959 \cdot 1224217 \cdot 31710071 \cdot 50588903$
2090681	7	$43 \cdot 1583 \cdot 2521 \cdot 12391 \cdot 25229 \cdot 33349 \cdot 97259 \cdot 122921 \cdot 3904447$
2091247	7	$911 \cdot 1481971 \cdot 4069801 \cdot 10024939 \cdot 33620959 \cdot 45165737$
2099711	7	$29 \cdot 43 \cdot 281 \cdot 1031423 \cdot 2022469 \cdot 2554819 \cdot 3263639 \cdot 14060593$
2110763	7	$13063 \cdot 378379 \cdot 2771693 \cdot 5800481 \cdot 25065013 \cdot 44400721$
2112419	7	$7 \cdot 2003 \cdot 44647 \cdot 6216841 \cdot 21762931 \cdot 25224053 \cdot 41591761$
2113399	7	$7 \cdot 29 \cdot 43 \cdot 35533 \cdot 97021 \cdot 148793 \cdot 189169 \cdot 1248101 \cdot 84283949$
2116801	7	$7 \cdot 197 \cdot 379 \cdot 617 \cdot 631 \cdot 8597 \cdot 20749 \cdot 30059 \cdot 6434429 \cdot 12815447$
2125451	7	$43 \cdot 239 \cdot 673 \cdot 701 \cdot 371029 \cdot 13882247 \cdot 60632573 \cdot 60888283$
2133811	7	$7 \cdot 29 \cdot 71 \cdot 197 \cdot 6301 \cdot 14533 \cdot 56701 \cdot 214607 \cdot 344177 \cdot 86683031$
2160997	7	$197 \cdot 883 \cdot 31249 \cdot 5364731 \cdot 8724619 \cdot 12604243 \cdot 31757839$
2223443	7	$449 \cdot 3347 \cdot 306139 \cdot 1279937 \cdot 1441133 \cdot 1894411 \cdot 75156859$
2289697	7	$281 \cdot 919969 \cdot 8282401 \cdot 13419673 \cdot 56015989 \cdot 89532059$
2299163	7	$38011 \cdot 259169 \cdot 659569 \cdot 5430923 \cdot 42765997 \cdot 97879993$
2371309	7	$463 \cdot 3067 \cdot 356287 \cdot 615679 \cdot 3672677 \cdot 11356591 \cdot 13685141$
2467447	7	$43 \cdot 673 \cdot 1667 \cdot 16759 \cdot 434113 \cdot 4654609 \cdot 11083759 \cdot 12463697$
2498939	7	$197 \cdot 491 \cdot 23297 \cdot 9553153 \cdot 15389977 \cdot 17070803 \cdot 43057393$
2522447	7	$71 \cdot 3851 \cdot 14939 \cdot 4860829 \cdot 9970423 \cdot 14705279 \cdot 88487491$
2522543	7	$29 \cdot 4957 \cdot 7351 \cdot 168491 \cdot 168869 \cdot 178613 \cdot 6155129 \cdot 7794557$
2550857	7	$7 \cdot 43 \cdot 239 \cdot 827 \cdot 1009 \cdot 2322797 \cdot 3777173 \cdot 7615441 \cdot 68688271$
2602673	7	$211 \cdot 1709 \cdot 117671 \cdot 2200843 \cdot 3193261 \cdot 13923617 \cdot 74859737$
2613979	7	$491 \cdot 104651 \cdot 114031 \cdot 259547 \cdot 2558683 \cdot 2881621 \cdot 28450871$
2708753	7	$43 \cdot 71 \cdot 113 \cdot 547 \cdot 22751 \cdot 393191 \cdot 624401 \cdot 7833659 \cdot 47840059$
2838217	7	$29 \cdot 113 \cdot 30059 \cdot 145643 \cdot 273043 \cdot 1756021 \cdot 1963081 \cdot 38710981$
2900297	7	$7 \cdot 29 \cdot 1009 \cdot 6917 \cdot 37507 \cdot 207061 \cdot 716143 \cdot 1198513 \cdot 63022681$
3051487	7	$29 \cdot 43 \cdot 19993 \cdot 125441 \cdot 581743 \cdot 2755369 \cdot 8442337 \cdot 19077073$
3056393	7	$29 \cdot 211 \cdot 547 \cdot 2689 \cdot 10487 \cdot 15331 \cdot 212479 \cdot 33385129 \cdot 79415617$
3088091	7	$1499 \cdot 2801 \cdot 7547 \cdot 1135247 \cdot 24173101 \cdot 25799593 \cdot 38655919$
3124843	7	$7 \cdot 127 \cdot 211 \cdot 438887 \cdot 2412341 \cdot 4046953 \cdot 28982843 \cdot 39969119$
3233441	7	$7 \cdot 71 \cdot 239 \cdot 2017 \cdot 2731 \cdot 3067 \cdot 7841 \cdot 891661 \cdot 3063257 \cdot 26591293$
3239963	7	$29 \cdot 71 \cdot 211 \cdot 5209 \cdot 4156643 \cdot 30066457 \cdot 45559193 \cdot 89773111$
3246073	7	$29 \cdot 197 \cdot 1395997 \cdot 1812959 \cdot 37535891 \cdot 37753829 \cdot 57095963$
3386909	7	$7 \cdot 211 \cdot 421 \cdot 803461 \cdot 2042237 \cdot 4908107 \cdot 13329653 \cdot 22612829$
3447673	7	$127 \cdot 211 \cdot 239 \cdot 6287 \cdot 6680521 \cdot 9218581 \cdot 14486221 \cdot 46751923$
3455317	7	$29 \cdot 967 \cdot 1163 \cdot 28631 \cdot 352633 \cdot 6653921 \cdot 26951597 \cdot 28820513$
3481007	7	$127 \cdot 449 \cdot 7211 \cdot 41357 \cdot 173573 \cdot 269543 \cdot 37372021 \cdot 59838143$
3487303	7	$7 \cdot 29 \cdot 13721 \cdot 516643 \cdot 1216559 \cdot 4960187 \cdot 11016587 \cdot 18801287$
3542243	7	$239 \cdot 421 \cdot 7547 \cdot 9871 \cdot 33923 \cdot 5464523 \cdot 37287559 \cdot 38128021$
3542927	7	$127 \cdot 281 \cdot 14813 \cdot 70141 \cdot 647557 \cdot 1287973 \cdot 3381743 \cdot 18911369$
3557251	7	$29 \cdot 25453 \cdot 47797 \cdot 66067 \cdot 125063 \cdot 273281 \cdot 472697 \cdot 53807629$
3589661	7	$43 \cdot 1289 \cdot 7393 \cdot 34721 \cdot 140281 \cdot 8216783 \cdot 10542253 \cdot 12375203$
3598873	7	$113 \cdot 21211 \cdot 48119 \cdot 3727459 \cdot 3738659 \cdot 13788881 \cdot 98035939$

p	r	$\Phi_r(p)$
3762137	7	$7 \cdot 29 \cdot 113 \cdot 127 \cdot 281 \cdot 743 \cdot 2107771 \cdot 6913229 \cdot 14425811 \cdot 22176337$
3863521	7	$29 \cdot 1723 \cdot 1301903 \cdot 7967107 \cdot 8714287 \cdot 9929263 \cdot 74163181$
3944953	7	$29 \cdot 43 \cdot 4336333 \cdot 9937229 \cdot 25858421 \cdot 44103641 \cdot 61506397$
3949919	7	$7 \cdot 2801 \cdot 36947 \cdot 279679 \cdot 337583 \cdot 983123 \cdot 1253911 \cdot 45042649$
4024961	7	$239 \cdot 10949 \cdot 25229 \cdot 160343 \cdot 200467 \cdot 462239 \cdot 637729 \cdot 6796763$
4480403	7	$8821 \cdot 70099 \cdot 208993 \cdot 734021 \cdot 1252483 \cdot 1369607 \cdot 49712419$
4505311	7	$29 \cdot 337 \cdot 379 \cdot 491 \cdot 2311 \cdot 2437 \cdot 8009 \cdot 547093 \cdot 3532999 \cdot 52742621$
4538603	7	$29 \cdot 127 \cdot 2129 \cdot 9199 \cdot 220529 \cdot 336211 \cdot 700127 \cdot 775349 \cdot 3010673$
4586077	7	$43 \cdot 71 \cdot 197 \cdot 21323 \cdot 59333 \cdot 107507 \cdot 251707 \cdot 4919671 \cdot 91842227$
4777793	7	$379 \cdot 10837 \cdot 134359 \cdot 2972803 \cdot 10304197 \cdot 23262709 \cdot 30248821$
4797857	7	$7 \cdot 71 \cdot 3221 \cdot 612067 \cdot 1045409 \cdot 12333203 \cdot 20644331 \cdot 46770809$
4809781	7	$43 \cdot 113 \cdot 12517 \cdot 66851 \cdot 4626511 \cdot 6883451 \cdot 8877499 \cdot 10770761$
4860269	7	$7 \cdot 659 \cdot 4201 \cdot 6553 \cdot 23857 \cdot 499157 \cdot 726923 \cdot 884171 \cdot 13561507$
4900547	7	$7 \cdot 1499 \cdot 4159 \cdot 32159 \cdot 181301 \cdot 25164287 \cdot 43296233 \cdot 49962179$
4988891	7	$1051 \cdot 9857 \cdot 14281 \cdot 1046711 \cdot 28467167 \cdot 45386657 \cdot 77058199$
5028323	7	$71 \cdot 113 \cdot 281 \cdot 8737 \cdot 12072649 \cdot 15074641 \cdot 58222277 \cdot 77445271$
5084423	7	$7 \cdot 379 \cdot 1499 \cdot 19979 \cdot 5995081 \cdot 17946349 \cdot 25171567 \cdot 80289007$
5198119	7	$43 \cdot 281 \cdot 3823 \cdot 33587 \cdot 1579579 \cdot 2488319 \cdot 35659037 \cdot 90721471$
5303383	7	$7 \cdot 43 \cdot 659 \cdot 743 \cdot 18523 \cdot 653899 \cdot 8876449 \cdot 33188513 \cdot 42308561$
5354207	7	$29 \cdot 17431 \cdot 1948619 \cdot 2796221 \cdot 3808141 \cdot 28914719 \cdot 77681983$
5357977	7	$29 \cdot 659 \cdot 2591 \cdot 36527 \cdot 2592199 \cdot 6900853 \cdot 9613129 \cdot 76068427$
5418277	7	$197023 \cdot 657959 \cdot 6573071 \cdot 28477093 \cdot 29668493 \cdot 35147309$
5517643	7	$29 \cdot 71 \cdot 1009 \cdot 24977 \cdot 84827 \cdot 162779 \cdot 751997 \cdot 2669507 \cdot 19617977$
5538097	7	$29 \cdot 43 \cdot 127 \cdot 631 \cdot 18047 \cdot 39103 \cdot 72689 \cdot 150991 \cdot 3284779 \cdot 11348093$
5635519	7	$7 \cdot 29 \cdot 659 \cdot 63659 \cdot 65479 \cdot 877577 \cdot 1980301 \cdot 2408281 \cdot 13725769$
5643667	7	$7 \cdot 757 \cdot 11080469 \cdot 11873471 \cdot 15349699 \cdot 30340507 \cdot 99521269$
5654071	7	$2927 \cdot 7547 \cdot 59333 \cdot 5776457 \cdot 9909593 \cdot 14802173 \cdot 29419237$
5696671	7	$7 \cdot 181889 \cdot 187139 \cdot 6003859 \cdot 15804461 \cdot 32822371 \cdot 46055129$
5710517	7	$7 \cdot 337 \cdot 12391 \cdot 226409 \cdot 708989 \cdot 6527641 \cdot 15974477 \cdot 70876499$
5848907	7	$7 \cdot 43 \cdot 1289 \cdot 1877 \cdot 2969 \cdot 4649 \cdot 185753 \cdot 423109 \cdot 3670969 \cdot 13804673$
5899363	7	$7 \cdot 43 \cdot 757 \cdot 3221 \cdot 22541 \cdot 66347 \cdot 10123709 \cdot 46354337 \cdot 81837659$
5996611	7	$29 \cdot 2633 \cdot 7211 \cdot 27077 \cdot 93871 \cdot 17097389 \cdot 23753129 \cdot 81810373$
6203371	7	$71 \cdot 883 \cdot 22247 \cdot 120947 \cdot 239611 \cdot 1724423 \cdot 15356881 \cdot 53238697$
6264107	7	$29 \cdot 2521 \cdot 17977 \cdot 37493 \cdot 284509 \cdot 5500153 \cdot 25258367 \cdot 31019927$
6414623	7	$3067 \cdot 59011 \cdot 190093 \cdot 246289 \cdot 2459801 \cdot 36326963 \cdot 92011039$
6450307	7	$7253 \cdot 32789 \cdot 33629 \cdot 46327 \cdot 251063 \cdot 331339 \cdot 901279 \cdot 2592829$
6459863	7	$43 \cdot 127 \cdot 197 \cdot 6203 \cdot 135017 \cdot 963047 \cdot 1548317 \cdot 4360091 \cdot 12405331$
6499631	7	$4663 \cdot 22247 \cdot 482441 \cdot 751871 \cdot 5595059 \cdot 18452113 \cdot 19406941$
6602399	7	$239 \cdot 1303 \cdot 2423 \cdot 33013 \cdot 53117 \cdot 86171 \cdot 312509 \cdot 472249 \cdot 4922681$
6835589	7	$113 \cdot 2801 \cdot 29387 \cdot 30871 \cdot 38459 \cdot 8529949 \cdot 16548211 \cdot 65442931$
6957107	7	$29 \cdot 71 \cdot 281 \cdot 337 \cdot 7211 \cdot 3943283 \cdot 7611283 \cdot 51182839 \cdot 52498139$
6990749	7	$379 \cdot 99023 \cdot 356749 \cdot 5165987 \cdot 8074529 \cdot 10425101 \cdot 20047189$
7144363	7	$449 \cdot 659 \cdot 1429 \cdot 4943 \cdot 6917 \cdot 10739 \cdot 146819 \cdot 169667 \cdot 171823 \cdot 200117$
7163917	7	$13903 \cdot 232877 \cdot 460013 \cdot 490169 \cdot 2888747 \cdot 3732499 \cdot 17172877$
7318039	7	$7 \cdot 43^2 \cdot 239 \cdot 4523 \cdot 174931 \cdot 186551 \cdot 280253 \cdot 33381587 \cdot 35957321$

p	r	$\Phi_r(p)$
7321123	7	$673 \cdot 3389 \cdot 7589 \cdot 8219 \cdot 83609 \cdot 5762597 \cdot 23817487 \cdot 94321109$
7434463	7	$7 \cdot 29 \cdot 43 \cdot 631 \cdot 1499 \cdot 2633 \cdot 5741 \cdot 20231 \cdot 34147 \cdot 27503813 \cdot 71203441$
7532381	7	$24179 \cdot 3786119 \cdot 15453887 \cdot 49118371 \cdot 49365779 \cdot 53241889$
7580773	7	$71 \cdot 127 \cdot 337 \cdot 421 \cdot 11117 \cdot 15541 \cdot 21673 \cdot 39439 \cdot 11804563 \cdot 85103131$
8104141	7	$19937 \cdot 34217 \cdot 729191 \cdot 1015561 \cdot 1699111 \cdot 5768869 \cdot 57211127$
8233787	7	$29 \cdot 449 \cdot 1163 \cdot 6091 \cdot 6581 \cdot 1598633 \cdot 6115061 \cdot 7036499 \cdot 7462547$
8297413	7	$29 \cdot 43 \cdot 113 \cdot 3361 \cdot 35141 \cdot 6164803 \cdot 11216717 \cdot 11364193 \cdot 24951991$
8383519	7	$29 \cdot 113 \cdot 281 \cdot 1667 \cdot 3221 \cdot 6343 \cdot 40699 \cdot 154351 \cdot 31461487 \cdot 56012111$
8399593	7	$43 \cdot 197 \cdot 379 \cdot 7589 \cdot 19559 \cdot 55721 \cdot 8675857 \cdot 21248221 \cdot 71744821$
8522749	7	$463 \cdot 1163 \cdot 46523 \cdot 223273 \cdot 262501 \cdot 2054851 \cdot 11239607 \cdot 11301893$
8561057	7	$7 \cdot 29 \cdot 1597 \cdot 11131 \cdot 148471 \cdot 429563 \cdot 3604609 \cdot 11776829 \cdot 40297139$
8638463	7	$7 \cdot 43 \cdot 211 \cdot 421 \cdot 2437 \cdot 39901 \cdot 102593 \cdot 960961 \cdot 16628389 \cdot 97493327$
8693947	7	$43 \cdot 2591 \cdot 18257 \cdot 20903 \cdot 25747 \cdot 159167 \cdot 355517 \cdot 1188601 \cdot 5864783$
8753207	7	$7 \cdot 43 \cdot 1499 \cdot 10627 \cdot 12923 \cdot 2138501 \cdot 2663459 \cdot 17963233 \cdot 70944889$
8816711	7	$7 \cdot 953 \cdot 28183 \cdot 358667 \cdot 4336333 \cdot 6995227 \cdot 13108103 \cdot 17518859$
8837557	7	$7 \cdot 71 \cdot 127 \cdot 43093 \cdot 785303 \cdot 857669 \cdot 5669441 \cdot 6260297 \cdot 7327139$
9194639	7	$43 \cdot 1051 \cdot 211933 \cdot 358499 \cdot 501187 \cdot 2049419 \cdot 2232931 \cdot 76726357$
9369317	7	$43 \cdot 4831 \cdot 42463 \cdot 413197 \cdot 591893 \cdot 1908817 \cdot 2453459 \cdot 66955771$
9961531	7	$113 \cdot 127 \cdot 3878519 \cdot 47990153 \cdot 59668603 \cdot 62095181 \cdot 98730787$
9997441	7	$1373 \cdot 20707 \cdot 1089383 \cdot 2475019 \cdot 8841449 \cdot 15675983 \cdot 93978403$
10024241	7	$127 \cdot 3851 \cdot 9661 \cdot 224071 \cdot 258637 \cdot 2742461 \cdot 28015093 \cdot 48228181$
10045507	7	$29 \cdot 239 \cdot 491 \cdot 12923 \cdot 127163 \cdot 275087 \cdot 1099729 \cdot 13689061 \cdot 44371111$
10225807	7	$71 \cdot 2647 \cdot 37199 \cdot 94949 \cdot 763267 \cdot 7842437 \cdot 10000733 \cdot 28773473$
10265509	7	$71 \cdot 197 \cdot 211 \cdot 3319 \cdot 4271 \cdot 57793 \cdot 73217509 \cdot 77179187 \cdot 85654213$
10393237	7	$7 \cdot 491 \cdot 6581 \cdot 10151 \cdot 56099 \cdot 133967 \cdot 168281 \cdot 52606471 \cdot 82508287$
10567813	7	$43 \cdot 127 \cdot 1303 \cdot 7127 \cdot 76819 \cdot 743849 \cdot 2500639 \cdot 4074757 \cdot 47171671$
10859309	7	$659 \cdot 23269 \cdot 46271 \cdot 71597 \cdot 74761 \cdot 1570339 \cdot 8582617 \cdot 32037461$
10869011	7	$113 \cdot 1471 \cdot 2003 \cdot 3361 \cdot 67607 \cdot 132833 \cdot 1684229 \cdot 2004787 \cdot 48588401$
10906559	7	$29 \cdot 883 \cdot 9199 \cdot 759739 \cdot 8829731 \cdot 8910679 \cdot 9742027 \cdot 12270301$
11041819	7	$43 \cdot 1429 \cdot 12853 \cdot 44647 \cdot 82601 \cdot 90371 \cdot 487481 \cdot 1787633 \cdot 7901251$
11171777	7	$7 \cdot 29 \cdot 197 \cdot 211 \cdot 673 \cdot 953 \cdot 1373 \cdot 2801 \cdot 8233 \cdot 82279 \cdot 4044503 \cdot 34094551$
11413789	7	$43 \cdot 393779 \cdot 4251647 \cdot 10500799 \cdot 10717631 \cdot 15261821 \cdot 17880241$
11467927	7	$449 \cdot 631 \cdot 1429 \cdot 1583 \cdot 2339 \cdot 2927 \cdot 17333 \cdot 497729 \cdot 1843997 \cdot 32586737$
11581567	7	$211 \cdot 953 \cdot 2801 \cdot 57667 \cdot 1998473 \cdot 27214867 \cdot 30591037 \cdot 44656991$
11725339	7	$127 \cdot 7757 \cdot 8093 \cdot 153889 \cdot 478171 \cdot 2856547 \cdot 24998849 \cdot 62028779$
11741893	7	$29 \cdot 379 \cdot 42407 \cdot 188707 \cdot 807731 \cdot 11020003 \cdot 51482383 \cdot 65021783$
11930839	7	$29 \cdot 43 \cdot 883 \cdot 6581 \cdot 134947 \cdot 2432711 \cdot 3294509 \cdot 8435309 \cdot 43627613$
12211543	7	$7 \cdot 29 \cdot 43 \cdot 197 \cdot 953 \cdot 4159 \cdot 9542891 \cdot 24530801 \cdot 30381947 \cdot 68407459$
12460403	7	$43 \cdot 71 \cdot 1373 \cdot 3067 \cdot 6791 \cdot 21911 \cdot 160357 \cdot 1403081 \cdot 1996933 \cdot 4354631$
12917647	7	$7 \cdot 743 \cdot 2311 \cdot 3347 \cdot 162499 \cdot 831503 \cdot 1611289 \cdot 8021749 \cdot 66130373$
13744979	7	$29 \cdot 2339 \cdot 189491 \cdot 418181 \cdot 533261 \cdot 1398209 \cdot 17813657 \cdot 94453577$
13850567	7	$967 \cdot 29723 \cdot 2421959 \cdot 3311519 \cdot 16288021 \cdot 22542479 \cdot 83410783$
13977811	7	$7 \cdot 43 \cdot 127 \cdot 2731 \cdot 805687 \cdot 6581849 \cdot 11615381 \cdot 18260663 \cdot 63515369$
13987403	7	$29 \cdot 71 \cdot 127 \cdot 75391 \cdot 109453 \cdot 540233 \cdot 2921339 \cdot 35989423 \cdot 61105087$
14125313	7	$337 \cdot 659 \cdot 3851 \cdot 17627 \cdot 6981563 \cdot 29563381 \cdot 38003239 \cdot 67172869$

p	r	$\Phi_r(p)$
14151341	7	$7 \cdot 43 \cdot 281 \cdot 2087 \cdot 6581 \cdot 30241 \cdot 258917 \cdot 3267727 \cdot 5631179 \cdot 47984021$
14228573	7	$547 \cdot 166363 \cdot 1405153 \cdot 7669411 \cdot 14274107 \cdot 17033311 \cdot 34800893$
14257819	7	$69623 \cdot 148457 \cdot 244301 \cdot 1530019 \cdot 3633029 \cdot 7253107 \cdot 82518283$
14323703	7	$29 \cdot 43 \cdot 113 \cdot 127 \cdot 6301 \cdot 325487 \cdot 582821 \cdot 1189651 \cdot 5654293 \cdot 60021529$
14409979	7	$29 \cdot 71 \cdot 113 \cdot 2857 \cdot 335917 \cdot 568807 \cdot 20740441 \cdot 53427683 \cdot 63614167$
14661061	7	$29 \cdot 211 \cdot 29401 \cdot 3128791 \cdot 7904653 \cdot 8871199 \cdot 11361113 \cdot 22145593$
14830391	7	$29 \cdot 43 \cdot 743 \cdot 1163 \cdot 13217 \cdot 88663 \cdot 538511 \cdot 1740187 \cdot 1808843 \cdot 4970659$
15225751	7	$491 \cdot 2591 \cdot 42337 \cdot 171403 \cdot 275087 \cdot 4882333 \cdot 14201839 \cdot 70752683$
15847499	7	$43 \cdot 1373 \cdot 2381 \cdot 15401 \cdot 164809 \cdot 343393 \cdot 2048621 \cdot 2231027 \cdot 28286441$
16044293	7	$43 \cdot 127 \cdot 379 \cdot 449 \cdot 242467 \cdot 9160537 \cdot 12199237 \cdot 12383659 \cdot 54702929$
16606193	7	$2591 \cdot 1339409 \cdot 1348747 \cdot 2516963 \cdot 2541701 \cdot 10134433 \cdot 69104869$
16847081	7	$29 \cdot 17851 \cdot 46439 \cdot 255361 \cdot 464927 \cdot 7237567 \cdot 18079181 \cdot 61220083$
16955431	7	$3319 \cdot 815809 \cdot 1018907 \cdot 3669373 \cdot 10842119 \cdot 11932439 \cdot 18142097$
18207781	7	$239 \cdot 337 \cdot 1373 \cdot 2689 \cdot 62903 \cdot 314077 \cdot 8550739 \cdot 9063097 \cdot 80032289$
18421549	7	$29 \cdot 827 \cdot 4271 \cdot 6763 \cdot 295513 \cdot 320083 \cdot 1177751 \cdot 6620587 \cdot 76488343$
18443773	7	$43 \cdot 71 \cdot 2591 \cdot 29947 \cdot 63799 \cdot 171403 \cdot 5629583 \cdot 36925813 \cdot 73099181$
18786679	7	$6637 \cdot 852881 \cdot 1694393 \cdot 3354037 \cdot 5747239 \cdot 6603437 \cdot 36010451$
18885121	7	$113 \cdot 211 \cdot 5153 \cdot 26573 \cdot 189127 \cdot 709913 \cdot 1602553 \cdot 2214983 \cdot 29155169$
18892483	7	$7 \cdot 449 \cdot 4691 \cdot 3471133 \cdot 16351007 \cdot 18604321 \cdot 37418599 \cdot 78056189$
18913991	7	$43 \cdot 127 \cdot 28477 \cdot 70393 \cdot 196337 \cdot 604031 \cdot 707071 \cdot 6333419 \cdot 7874819$
19646813	7	$71 \cdot 10711 \cdot 35491 \cdot 79031 \cdot 3943661 \cdot 7700911 \cdot 14982829 \cdot 59253377$
20365361	7	$29 \cdot 113 \cdot 13063 \cdot 37423 \cdot 65899 \cdot 308141 \cdot 1545433 \cdot 17212931 \cdot 82444447$
20605183	7	$29 \cdot 1009 \cdot 241739 \cdot 950251 \cdot 2212631 \cdot 4046239 \cdot 29253001 \cdot 43476413$
21101371	7	$1471 \cdot 25621 \cdot 29723 \cdot 38767 \cdot 126757 \cdot 15828457 \cdot 20260003 \cdot 50009261$
21999787	7	$43 \cdot 3767 \cdot 29527 \cdot 96293 \cdot 8901019 \cdot 18195577 \cdot 18254237 \cdot 83265197$
22194313	7	$7 \cdot 43 \cdot 1877 \cdot 17921 \cdot 116047 \cdot 2935213 \cdot 22445249 \cdot 30195887 \cdot 51134203$
22308911	7	$29 \cdot 1429 \cdot 2017 \cdot 33811 \cdot 39383 \cdot 49169 \cdot 12581689 \cdot 20851321 \cdot 85862477$
22444277	7	$659 \cdot 2591 \cdot 19237 \cdot 619921 \cdot 4299653 \cdot 9368059 \cdot 12089351 \cdot 12892027$
22489241	7	$281 \cdot 6917 \cdot 9787 \cdot 33013 \cdot 33811 \cdot 160091 \cdot 550369 \cdot 1385749 \cdot 49903001$
22853483	7	$197 \cdot 617 \cdot 6917 \cdot 8933 \cdot 26153 \cdot 275521 \cdot 2499953 \cdot 14378029 \cdot 73238677$
22951283	7	$29 \cdot 1597 \cdot 2339 \cdot 141961 \cdot 202567 \cdot 234571 \cdot 1711753 \cdot 8649467 \cdot 13510337$
23150119	7	$50821 \cdot 638359 \cdot 1123403 \cdot 2347549 \cdot 4149881 \cdot 20643421 \cdot 21001177$
23893139	7	$197 \cdot 3739 \cdot 75377 \cdot 1032697 \cdot 1187551 \cdot 4646671 \cdot 14660521 \cdot 40110883$
24066689	7	$29 \cdot 71 \cdot 463 \cdot 6217 \cdot 139301 \cdot 4508687 \cdot 12681593 \cdot 55084261 \cdot 74726429$
25032031	7	$43 \cdot 337 \cdot 1303 \cdot 1583 \cdot 4831 \cdot 9157 \cdot 1525763 \cdot 1818293 \cdot 3183559 \cdot 21066809$
27287233	7	$7 \cdot 43 \cdot 113 \cdot 617 \cdot 631 \cdot 13469 \cdot 17921 \cdot 21701 \cdot 1564417 \cdot 38480653 \cdot 98861057$
27568511	7	$239 \cdot 3557 \cdot 7477 \cdot 11299 \cdot 31193 \cdot 464213 \cdot 2266237 \cdot 3274811 \cdot 56880671$
28208827	7	$71 \cdot 491 \cdot 5783 \cdot 15401 \cdot 489133 \cdot 523573 \cdot 3009203 \cdot 4658557 \cdot 45202361$
28234649	7	$1163 \cdot 8807 \cdot 161561 \cdot 500459 \cdot 1078757 \cdot 2027411 \cdot 6028457 \cdot 46399151$
28361629	7	$7477 \cdot 292531 \cdot 640949 \cdot 3785531 \cdot 29562317 \cdot 54942413 \cdot 60379747$
28379903	7	$1051 \cdot 2143 \cdot 11117 \cdot 645877 \cdot 685063 \cdot 15504413 \cdot 46640651 \cdot 65215781$
28578083	7	$1362551 \cdot 27985189 \cdot 46422433 \cdot 53190257 \cdot 66210901 \cdot 87383227$
29211373	7	$43 \cdot 743 \cdot 1583 \cdot 3613 \cdot 367739 \cdot 669173 \cdot 8397901 \cdot 17979781 \cdot 91510819$
29609431	7	$29 \cdot 127 \cdot 463 \cdot 497281 \cdot 704131 \cdot 1087717 \cdot 1562107 \cdot 24689519 \cdot 26903143$
29695933	7	$7 \cdot 29 \cdot 43 \cdot 5573 \cdot 15443 \cdot 116201 \cdot 809173 \cdot 5336563 \cdot 21595351 \cdot 84240997$

p	r	$\Phi_r(p)$
30314399	7	$15443 \cdot 199697 \cdot 2437219 \cdot 2907577 \cdot 5152463 \cdot 69163459 \cdot 99649061$
30693889	7	$43 \cdot 379 \cdot 463 \cdot 2017 \cdot 6091 \cdot 168617 \cdot 1045549 \cdot 1093331 \cdot 1261387 \cdot 37100743$
30823421	7	$2003 \cdot 6553 \cdot 25579 \cdot 249859 \cdot 2857709 \cdot 3456377 \cdot 14080837 \cdot 73505153$
31029787	7	$43 \cdot 127 \cdot 337 \cdot 1289 \cdot 15443 \cdot 55931 \cdot 117937 \cdot 7935047 \cdot 19252423 \cdot 24179569$
31307207	7	$7 \cdot 29 \cdot 197 \cdot 60397 \cdot 84673 \cdot 164599 \cdot 1302449 \cdot 1853713 \cdot 2389591 \cdot 4848299$
31839151	7	$7 \cdot 43 \cdot 281 \cdot 379 \cdot 5573 \cdot 8807 \cdot 28463 \cdot 120401 \cdot 428639 \cdot 15778337 \cdot 28567757$
32244859	7	$29 \cdot 281 \cdot 3137 \cdot 26293 \cdot 136949 \cdot 3212413 \cdot 10808771 \cdot 13802909 \cdot 25478083$
33729187	7	$65003 \cdot 240899 \cdot 662369 \cdot 3182341 \cdot 21742757 \cdot 27410237 \cdot 74850161$
33804481	7	$71 \cdot 127 \cdot 211 \cdot 281 \cdot 11159 \cdot 212297 \cdot 496427 \cdot 4835153 \cdot 21772969 \cdot 22544593$
35236219	7	$43 \cdot 2647 \cdot 370133 \cdot 379849 \cdot 5516827 \cdot 16272901 \cdot 23886157 \cdot 55775567$
35893283	7	$239 \cdot 162499 \cdot 269851 \cdot 377231 \cdot 583031 \cdot 609407 \cdot 34740581 \cdot 43819049$
37620101	7	$7 \cdot 29 \cdot 113 \cdot 169373 \cdot 198073 \cdot 202567 \cdot 271181 \cdot 2365399 \cdot 2422421 \cdot 11702909$
37843849	7	$7 \cdot 127 \cdot 379289 \cdot 10479673 \cdot 11367077 \cdot 16158563 \cdot 61939991 \cdot 73068367$
38475977	7	$7 \cdot 29 \cdot 197 \cdot 547 \cdot 953 \cdot 2843 \cdot 93871 \cdot 2171737 \cdot 5175493 \cdot 7029611 \cdot 7380689$
38859413	7	$71 \cdot 197 \cdot 463 \cdot 96167 \cdot 137369 \cdot 2263381 \cdot 3597749 \cdot 66699137 \cdot 74105137$
39434663	7	$3557 \cdot 3613 \cdot 8387 \cdot 14281 \cdot 19181 \cdot 292223 \cdot 832721 \cdot 22835639 \cdot 22922047$
39890003	7	$43 \cdot 673 \cdot 2017 \cdot 4019 \cdot 27329 \cdot 114157 \cdot 223273 \cdot 697397 \cdot 1695611 \cdot 20850103$
40106387	7	$29 \cdot 8233 \cdot 236293 \cdot 948053 \cdot 2849351 \cdot 9070559 \cdot 39311917 \cdot 76583893$
40178219	7	$29 \cdot 659 \cdot 6329 \cdot 388991 \cdot 516839 \cdot 1106029 \cdot 4427417 \cdot 5061281 \cdot 6979967$
40188523	7	$43 \cdot 127 \cdot 631 \cdot 1429 \cdot 1493759 \cdot 4569811 \cdot 28565671 \cdot 56512723 \cdot 77644211$
41111381	7	$29^2 \cdot 2843 \cdot 3823 \cdot 6217 \cdot 500333 \cdot 838153 \cdot 1867951 \cdot 1881811 \cdot 57635551$
41442329	7	$29 \cdot 43 \cdot 379 \cdot 1289 \cdot 61979 \cdot 748567 \cdot 894419 \cdot 950867 \cdot 6640663 \cdot 31736489$
41981081	7	$3221 \cdot 19237 \cdot 140057 \cdot 503231 \cdot 2538803 \cdot 3414181 \cdot 8413021 \cdot 17189131$
42075751	7	$127 \cdot 2003 \cdot 35393 \cdot 54139 \cdot 243517 \cdot 390391 \cdot 3172471 \cdot 5671499 \cdot 6655097$
42523937	7	$7 \cdot 6917 \cdot 7127 \cdot 63617 \cdot 169709 \cdot 587063 \cdot 1477043 \cdot 30176749 \cdot 60652663$
42560369	7	$113 \cdot 127 \cdot 281 \cdot 1597 \cdot 3739 \cdot 12503 \cdot 14813 \cdot 43051 \cdot 281233 \cdot 3492077 \cdot 31520483$
43305763	7	$211 \cdot 379 \cdot 2591 \cdot 58031 \cdot 111833 \cdot 185123 \cdot 8455147 \cdot 32955749 \cdot 95091641$
44316197	7	$239 \cdot 281 \cdot 42239 \cdot 72101 \cdot 375509 \cdot 940003 \cdot 2141749 \cdot 6141059 \cdot 7977271$
44894383	7	$1933 \cdot 14869 \cdot 26209 \cdot 150893 \cdot 702913 \cdot 27240151 \cdot 41178523 \cdot 91355993$
46242337	7	$7 \cdot 71 \cdot 631 \cdot 659 \cdot 3347 \cdot 34763 \cdot 121577 \cdot 298733 \cdot 313909 \cdot 698111 \cdot 51089501$
46268653	7	$29 \cdot 701 \cdot 1303 \cdot 16871 \cdot 169177 \cdot 2642333 \cdot 15521101 \cdot 46904327 \cdot 67461437$
47971057	7	$7 \cdot 197 \cdot 4019 \cdot 16339 \cdot 1534219 \cdot 2120917 \cdot 30030043 \cdot 33295109 \cdot 41363813$
48298753	7	$29 \cdot 43 \cdot 421 \cdot 897373 \cdot 2083957 \cdot 2444569 \cdot 2787443 \cdot 23082067 \cdot 82209541$
49854929	7	$43 \cdot 883 \cdot 24977 \cdot 77267 \cdot 516433 \cdot 1036267 \cdot 1126343 \cdot 8647913 \cdot 40199209$
49937521	7	$113 \cdot 4691 \cdot 10711 \cdot 31333 \cdot 48091 \cdot 574393 \cdot 3163469 \cdot 19285169 \cdot 51728041$
50058647	7	$29^2 \cdot 5783 \cdot 78989 \cdot 2830073 \cdot 4965563 \cdot 7082377 \cdot 17306381 \cdot 23779757$
50837341	7	$1163 \cdot 2843 \cdot 23899 \cdot 30059 \cdot 56393 \cdot 1055881 \cdot 3963317 \cdot 5527481 \cdot 5571343$
51418093	7	$29 \cdot 631 \cdot 3137 \cdot 6917 \cdot 9157 \cdot 166027 \cdot 384259 \cdot 787879 \cdot 4589999 \cdot 22029673$
51706331	7	$43 \cdot 211 \cdot 379 \cdot 279511 \cdot 1264649 \cdot 2441293 \cdot 7497043 \cdot 20739853 \cdot 41417867$
54797191	7	$7 \cdot 71 \cdot 757 \cdot 1093 \cdot 2689 \cdot 102551 \cdot 4918411 \cdot 20991811 \cdot 31632763 \cdot 73102933$
55067269	7	$113 \cdot 1723 \cdot 2549 \cdot 195161 \cdot 579629 \cdot 882631 \cdot 2717821 \cdot 4843693 \cdot 42747223$
55739507	7	$421 \cdot 1429 \cdot 11173 \cdot 59921 \cdot 78653 \cdot 411083 \cdot 3030371 \cdot 26621561 \cdot 28545749$
55959779	7	$7 \cdot 29 \cdot 2689 \cdot 5867 \cdot 53089 \cdot 60383 \cdot 219437 \cdot 10421951 \cdot 14671147 \cdot 89147843$
56260723	7	$7 \cdot 14057 \cdot 68881 \cdot 87641 \cdot 96601 \cdot 853091 \cdot 1161007 \cdot 6951883 \cdot 80263877$
56414999	7	$29 \cdot 71 \cdot 54881 \cdot 158621 \cdot 276277 \cdot 372107 \cdot 4841593 \cdot 39829987 \cdot 90723011$

p	r	$\Phi_r(p)$
57038833	7	$43 \cdot 113 \cdot 15877 \cdot 54979 \cdot 103307 \cdot 4871861 \cdot 5050417 \cdot 50920381 \cdot 62728961$
58284521	7	$7 \cdot 125399 \cdot 394409 \cdot 445019 \cdot 2522087 \cdot 21093199 \cdot 63569549 \cdot 75240257$
58776539	7	$43 \cdot 1471 \cdot 11411 \cdot 150893 \cdot 155821 \cdot 920473 \cdot 5183221 \cdot 6154751 \cdot 82737593$
59329663	7	$7 \cdot 113 \cdot 2843 \cdot 4271 \cdot 36037 \cdot 136403 \cdot 2823857 \cdot 3085741 \cdot 7503763 \cdot 14128451$
59708479	7	$127 \cdot 967 \cdot 7001 \cdot 25117 \cdot 381739 \cdot 546967 \cdot 4843693 \cdot 22607411 \cdot 91770463$
59903947	7	$337 \cdot 3361 \cdot 21491 \cdot 74131 \cdot 99191 \cdot 443689 \cdot 2097257 \cdot 16206233 \cdot 17119579$
61118989	7	$7 \cdot 4243 \cdot 4831 \cdot 69497 \cdot 462911 \cdot 1901131 \cdot 3386741 \cdot 19301059 \cdot 90867827$
61762193	7	$29 \cdot 281 \cdot 463 \cdot 757 \cdot 2087 \cdot 3011 \cdot 259813 \cdot 1083881 \cdot 1155127 \cdot 2018899 \cdot 4709069$
61964431	7	$2521 \cdot 9829 \cdot 168869 \cdot 787879 \cdot 1712383 \cdot 2880739 \cdot 57176701 \cdot 60875039$
64632527	7	$7 \cdot 29 \cdot 71 \cdot 113 \cdot 449 \cdot 107857 \cdot 237707 \cdot 1054649 \cdot 2712179 \cdot 14250167 \cdot 95387419$
64687097	7	$29 \cdot 43 \cdot 7127 \cdot 85751 \cdot 1098511 \cdot 1147301 \cdot 10887367 \cdot 81337649 \cdot 86138501$
66606157	7	$281 \cdot 5153 \cdot 6301 \cdot 26293 \cdot 38977 \cdot 41609 \cdot 50821 \cdot 502699 \cdot 666821 \cdot 13173889$
68113807	7	$10613 \cdot 30829 \cdot 83791 \cdot 574813 \cdot 3701881 \cdot 10213001 \cdot 12842411 \cdot 13051697$
68441137	7	$71 \cdot 28001 \cdot 62539 \cdot 3685501 \cdot 4812851 \cdot 18304511 \cdot 26277259 \cdot 96890977$
71014399	7	$7 \cdot 197 \cdot 281 \cdot 1093 \cdot 10753 \cdot 56533 \cdot 6121543 \cdot 17289007 \cdot 59463251 \cdot 79154657$
72331219	7	$43 \cdot 28547 \cdot 644197 \cdot 2722931 \cdot 8324471 \cdot 8465297 \cdot 22430381 \cdot 42076049$
73553561	7	$29 \cdot 24473 \cdot 29023 \cdot 33013 \cdot 39971 \cdot 3750839 \cdot 4043341 \cdot 4434571 \cdot 86625211$
74306809	7	$3851 \cdot 38921 \cdot 604031 \cdot 662719 \cdot 1496167 \cdot 3987229 \cdot 15379967 \cdot 30578689$
75682331	7	$29 \cdot 113 \cdot 2143 \cdot 136963 \cdot 2045359 \cdot 4599533 \cdot 12094321 \cdot 34224121 \cdot 50172767$
75790223	7	$113 \cdot 3347 \cdot 17599 \cdot 27847 \cdot 288583 \cdot 509363 \cdot 11523359 \cdot 13190311 \cdot 45766351$
76085351	7	$29 \cdot 127 \cdot 17333 \cdot 92107 \cdot 1504651 \cdot 1612759 \cdot 5960809 \cdot 36097979 \cdot 63189491$
77118799	7	$113 \cdot 275339 \cdot 532267 \cdot 1731731 \cdot 2435203 \cdot 9050329 \cdot 9605737 \cdot 34647761$
77580787	7	$29 \cdot 281 \cdot 491 \cdot 659 \cdot 2297 \cdot 6917 \cdot 9437 \cdot 804161 \cdot 1985677 \cdot 3533293 \cdot 97748449$
77720411	7	$71 \cdot 113 \cdot 7687 \cdot 108739 \cdot 1192171 \cdot 4204523 \cdot 5063129 \cdot 15639079 \cdot 82803001$
77827181	7	$197 \cdot 337 \cdot 1877 \cdot 138923 \cdot 400681 \cdot 2568119 \cdot 7625227 \cdot 39191363 \cdot 41744207$
78106307	7	$281 \cdot 1289 \cdot 36779 \cdot 332473 \cdot 949621 \cdot 1351099 \cdot 1483763 \cdot 2085287 \cdot 12913181$
85891943	7	$71 \cdot 337 \cdot 911 \cdot 839791 \cdot 1201201 \cdot 2164121 \cdot 2627143 \cdot 43202587 \cdot 74344019$
87885949	7	$491 \cdot 631 \cdot 12853 \cdot 225569 \cdot 791519 \cdot 1592753 \cdot 2320207 \cdot 2597113 \cdot 67529659$
88761593	7	$43 \cdot 71 \cdot 239 \cdot 911 \cdot 6763 \cdot 61153 \cdot 70589 \cdot 129403 \cdot 1190897 \cdot 6944813 \cdot 23546923$
89669737	7	$239 \cdot 757 \cdot 1855981 \cdot 4543813 \cdot 10344223 \cdot 12682027 \cdot 26326301 \cdot 98653213$
90107203	7	$197 \cdot 421 \cdot 39971 \cdot 79997 \cdot 145601 \cdot 4639979 \cdot 5009327 \cdot 9937789 \cdot 60012331$
90196549	7	$29 \cdot 239 \cdot 1093 \cdot 8429 \cdot 39047 \cdot 60103 \cdot 105211 \cdot 7074523 \cdot 53210123 \cdot 90721667$
92892127	7	$29 \cdot 1093 \cdot 1933 \cdot 18439 \cdot 748609 \cdot 13756877 \cdot 25182221 \cdot 35846021 \cdot 61175311$
93058573	7	$10459 \cdot 16927 \cdot 399281 \cdot 512597 \cdot 1225183 \cdot 9568931 \cdot 35313391 \cdot 43292201$
95303477	7	$29 \cdot 281 \cdot 673 \cdot 967 \cdot 16493 \cdot 159097 \cdot 1333193 \cdot 11940041 \cdot 41532961 \cdot 81442901$
95709899	7	$127 \cdot 3011 \cdot 5503 \cdot 11047 \cdot 11971 \cdot 60271 \cdot 130579 \cdot 1964243 \cdot 3893191 \cdot 45895459$
96397919	7	$7001 \cdot 35533 \cdot 314693 \cdot 627901 \cdot 5457971 \cdot 5471887 \cdot 9429869 \cdot 57964453$
97023863	7	$71 \cdot 7309 \cdot 21169 \cdot 33461 \cdot 325487 \cdot 1758947 \cdot 3357901 \cdot 13230953 \cdot 89221679$
97073359	7	$967 \cdot 2017 \cdot 29723 \cdot 109201 \cdot 213599 \cdot 288317 \cdot 2990051 \cdot 20861779 \cdot 34407199$
97185839	7	$29 \cdot 71 \cdot 11593 \cdot 189743 \cdot 570851 \cdot 13680311 \cdot 19892419 \cdot 29525861 \cdot 40559359$
97330193	7	$29 \cdot 113 \cdot 1093 \cdot 810643 \cdot 895231 \cdot 4341191 \cdot 26141809 \cdot 34000723 \cdot 84760103$
97330283	7	$7 \cdot 43 \cdot 113 \cdot 7477 \cdot 9437 \cdot 24977 \cdot 42379 \cdot 1958419 \cdot 2292949 \cdot 6886727 \cdot 10821259$
99129991	7	$29 \cdot 44549 \cdot 220151 \cdot 3705101 \cdot 15917371 \cdot 19970147 \cdot 29313551 \cdot 96639061$

A.6 Programs

A.6.1 cubeprg.gp

```
\\
\\  cubeprg.gp
\\
\\  by Takeshi GOTO (Oct. 2005)
\\    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
\\
\\  used to find all pairs (P,Q,R) satisfying
\\    Q^3 | \Phi_R(P), P,Q<10^8, 6680<=R<5 \times 10^7.
\\  output files: mysqrs.txt, mycubes.txt, numchkd.txt
\\
prmdiv(a)=
{
  local(p);
  p=2;
  while(p<=a,
    if(a%p==0,return(p));
    p=nextprime(p+1)
  )
}

modpow(a,b,p)=
{
  local(x);
  x=Mod(a,p)^b;
  return(lift(x))
}

fngen(q)=
{
  local(a,qq,p,c);
  a=2; c=0;
  while(c==0,
    qq=q-1;c=1;
    while(qq>1,
      p=prmdiv(qq);
      if(modpow(a,(q-1)/p,q)==1,c=0);
      while(qq%p==0,qq=qq/p)
    );
    if(c==0,a=a+1)
  );
  if(modpow(a,q-1,q^2)==1,a=a+q);
  return(a)
}
```

```
check(f,t)=
{
  local(a,g,q,r);
  r=nextprime(f);
  if(r>t,return("Not a valid interval."));
  while(r<=t,
    print("Checking for R= ",r);
    q=2*r+1;
    while(q<10^8,
      if(isprime(q),
        g=Mod(fngen(q),q^2);
        a=g^(q*(q-1)/r); p=1;
        for(i=1,r-1,
          p=lift(p*a);
          if(p<10^2, write("mysqrs.txt",q,"^2 divides ",p,"^",r,"-1"),
            if(p<10^8 && isprime(p) && modpow(p,r,q^3)==1,
              print(q,"^3 divides ",p,"^",r,"-1");
              write("mycubes.txt",q,"^3 divides ",p,"^",r,"-1")
            )
          )
        )
      );
      q=q+2*r
    );
    r=nextprime(r+1)
  );
  write("numchkd.txt","Checked R= ",f," to R= ",t)
}
```

A.6.2 cubeprg.ub

```
 10   '
 20   '  CUBEPRG.UB
 30   '
 40   '  originally produced by Paul M. Jenkins
 50   '  modified by Takeshi GOTO (Oct. 2005)
 60   '    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
 70   '
 80   '  This file enable us to find all pairs (P,Q,R) satisfying
 90   '    Q^3 | \Phi_R(P), P,Q<10^8, 6680<=R<5 \times 10^7.
100   '  We did it, using GP script "cubeprg.gp".
110   '  output files: MYSQRS.TXT, MYCUBES.TXT, NUMCHKD.TXT
120   '  expected CPU time: about two hours for an interval of length 10^4
130   '
140   input "Begin checking at";F
```

```
150  input "Up to";T
160  clr time
170  R=nxtprm(F-1)
180  if R>T then print "Not a valid interval":end
190  print "Checking for R=";R
200  Q=2*R+1
210  while Q<100000000
220    if prmdiv(Q)<Q then goto 380
230    G=fnGEN(Q)
240    A=modpow(G,Q*(Q-1)//R,Q^2):P=1
250    for I=1 to R-1
260      P=(P*A)@(Q^2) 'Note that if R>5000, then Q^2>10^8
270      if P<100 then
280        :print=print+"mysqrs.txt"
290        :print Q;"^2 divides";P;"^";R;"-1"
300        :print=print
310      endif
320      if and{P<100000000,prmdiv(P)=P,modpow(P,R,Q^3)=1} then
330        :print=print+"mycubes.txt"
340        :print Q;"^3 divides";P;"^";R;"-1"
350        :print=print
360      endif
370    next
380    Q=Q+2*R
390  wend
400  R=nxtprm(R):if R<=T then goto 190
410  print=print+"numchkd.txt"
420  print "Checked R=";F;"to R=";T;"in";time
430  print=print
440  end
450  '
460  '  a generator of (Z/Q^2Z)^*
470  '
480  fnGEN(Q)
490  local A,QQ,P
500  A=2
510  QQ=Q-1
520  while QQ>1
530    P=prmdiv(QQ)
540    if modpow(A,(Q-1)//P,Q)=1 then A=A+1:goto 510
550    while QQ@P=0:QQ=QQ//P:wend
560  wend
570  if modpow(A,Q-1,Q^2)=1 then A=A+Q
580  return(A)
```

A.6.3 sqrprg.ub

```
10   '
20   '  SQRPRG.UB
30   '
40   '  originally produced by Paul M. Jenkins
50   '  modified by Takeshi GOTO (Oct. 2005)
60   '    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
70   '
80   '  used to check for cubic or square divisors of \Phi_R(P)
90   '    for 2503<R<=6679
100  '  output files: MYCUBES.TXT, NUMCHKD.TXT
110  '  expected CPU time: about an hour for 2503<R<=6679
120  '
130  input "Begin checking at";F
140  input "Up to";T:clr time
150  open "memo" as file1(10000) word 100:C=1
160  R=nxtprm(F-1)
170  if R>T then print "Not a valid interval":end
180  print "Checking for R=";R
190  Q=2*R+1:C=1
200  while Q<10^8
210    if prmdiv(Q)<Q then goto 460
220    G=fnGEN(Q)
230    A=modpow(G,Q*(Q-1)//R,Q^2):P=1
240    for I=1 to R-1
250      P=(P*A)@(Q^2)
260      while P<10^8
270        if or{prmdiv(P)<P,P<10^2} then goto 430
280        for I=1 to C-1
290          if P=file1(I) then
300            :print=print+"mysqrs.txt"
310            :print "More than one square divides";P;"^";R;"-1"
320            :print=print
330            :cancel for
340            :goto 370
350          endif
360        next
370        file1(C)=P:C=C+1
380        if modpow(P,R,Q^3)=1 then
390          :print=print+"mycubes.txt"
400          :print Q;"^3 divides";P;"^";R;"-1"
410          :print=print
420        endif
430        P=P+Q^2
440      wend
450    next
```

```
460     Q=Q+2*R
470   wend
480   R=nxtprm(R):if R<=T then goto 180
490   print=print+"numchkd.txt"
500   print "Checked R=";F;"to R=";T;"in";time
510   print=print
520   close:kill "memo+"
530   end
540   '
550   '  a generator of (Z/Q^2Z)^*
560   '
570   fnGEN(Q)
580   local A,QQ,P
590   A=2
600   QQ=Q-1
610   while QQ>1
620     P=prmdiv(QQ)
630     if modpow(A,(Q-1)//P,Q)=1 then A=A+1:goto 600
640     while QQ@P=0:QQ=QQ//P:wend
650   wend
660   if modpow(A,Q-1,Q^2)=1 then A=A+Q
670   return(A)
```

A.6.4 rvalue.ub

```
 10   '
 20   '  RVALUE.UB
 30   '
 40   '  originally produced by Paul M. Jenkins as "Prop8.ub"
 50   '  modified by Takeshi GOTO (Oct. 2005)
 60   '    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
 70   '
 80   '  used to find the values R(P) for P<100
 90   '  output file: RVALUE.TXT
100   '  expected CPU time: about thirty minutes
110   '
120   Epsilon=10^5*#Eps ' an upper bound of round-off error
130   P=3
140   print=print+"rvalue.txt"
150   while P<100
160     R=4500
170     while R<5*10^4
180       Q=1
190       C=8*log(10)+log(R)
200       while Q<10^8
210         Q=Q+2*R
```

```
220       if prmdiv(Q)=Q then C=C+log(Q)
230     wend
240     if C+Epsilon>(R-1)*log(P) then
250       :print R
260       :if C<=(R-1)*log(P) then print "check again"
270     endif
280     R=nxtprm(R)
290   wend
300   print "for P=";P
310   P=nxtprm(P)
320 wend
330 print=print
```

A.6.5 claim1.ub

```
10 ’
20 ’  CLAIM1.UB
30 ’
40 ’  originally produced by Paul M. Jenkins as "Prop5.ub"
50 ’  modified by Takeshi GOTO (Oct. 2005)
60 ’    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
70 ’
80 ’  used to show Claim1
90 ’  expected CPU time: about one minute
100 ’
110 Epsilon=10^5*#eps ’ an upper bound of round-off error
120 R=6007
130 while R<5*10^4
140   Q=1
150   C=8*log(10)+0.5*log(R)
160   while Q<10^8
170     Q=Q+2*R
180     if prmdiv(Q)=Q then C=C+log(Q)
190   wend
200   if C+Epsilon>(R-1)*log(10) then print R
210   R=nxtprm(R)
220 wend
```

A.6.6 claim2.ub

```
10 ’
20 ’  CLAIM2.UB
30 ’
40 ’  originally produced by Paul M. Jenkins as "Prop6.ub"
50 ’  modified by Takeshi GOTO (Oct. 2005)
60 ’    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
```

```
70   '
80   '  used to show Claim2
90   '  expected CPU time: less than one minute
100  '
110  Epsilon=10^5*#eps ' an upper bound of round-off error
120  R=nxtprm(4000)
130  while R<6680
140    Q=1
150    C=8*log(10)+log(R)
160    while Q<10^8
170      Q=Q+2*R
180      if prmdiv(Q)=Q then C=C+log(Q)
190    wend
200    if C+Epsilon>2*(R-1)*log(10) then print R
210    R=nxtprm(R)
220  wend
```

A.6.7 claim3.ub

```
10   '
20   '  CLAIM3.UB
30   '
40   '  originally produced by Paul M. Jenkins as "Prop6.ub"
50   '  modified by Takeshi GOTO (Oct. 2005)
60   '    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
70   '
80   '  used to show Claim3
90   '  expected CPU time: less than one minute
100  '
110  Epsilon=10^5*#eps ' an upper bound of round-off error
120  R=nxtprm(2500)
130  while R<4724
140    Q=1
150    C=16*log(10)+log(R)
160    while Q<10^8
170      Q=Q+2*R
180      if prmdiv(Q)=Q then C=C+log(Q)
190    wend
200    if C+Epsilon>6*(R-1)*log(10) then print R
210    R=nxtprm(R)
220  wend
```

A.6.8 claim4.ub

```
10   '
20   '  CLAIM4.UB
```

```
30  '
40  '  originally produced by Paul M. Jenkins as "Prop6.ub"
50  '  modified by Takeshi GOTO (Oct. 2005)
60  '    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
70  '
80  '  used to show Claim4
90  '  expected CPU time: less than one minute
100 '
110 Epsilon=10^5*#eps ' an upper bound of round-off error
120 R=nxtprm(2000)
130 while R<2708
140   Q=1
150   C=16*log(10)+log(R)
160   while Q<10^8
170     Q=Q+2*R
180     if prmdiv(Q)=Q then C=C+log(Q)
190   wend
200   if C+Epsilon>7*(R-1)*log(10) then print R
210   R=nxtprm(R)
220 wend
```

A.6.9 accept.ub

```
10  '
20  '  ACCEPT.UB
30  '
40  '  by Takeshi GOTO (Oct. 2005)
50  '    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
60  '
70  '  used to find acceptable values \Phi_R(P) for 7<=R<=4723
80  '  output file: ACCEPT.TXT
90  '  expected CPU time: about three hours for one value of R
100 '
110 input "Begin checking at";F
120 input "Up to";T
130 R=nxtprm(F-1)
140 if R>T then print "Not a valid interval":end
150 print
160 '
170 clr time:clr C:clr C1
180 print=print+"accept.txt"
190 print "Checking for R=";R
200 print=print
210 '
220 '  write all pairs (P,Q) satisfying Q | \Phi_R(P) and P,Q<10^8
230 '  on file1
```

```
240 open "pairs.ubd" for create as file1(2^32-1) word 8
250 Q=2*R+1
260 while Q<100000000
270   if prmdiv(Q)<Q then goto 400
280   print "                         ";chr(13);
290   print " Q=";Q;
300   G=fnGEN(Q)
310   W=modpow(G,(Q-1)//R,Q):P=1
320   for I%=1 to R-1
330     P=(P*W)@Q
340     while P<100000000
350       if or{prmdiv(P)<P,P=2} then goto 370
360       C=C+1:file1(C)=pack(P,Q)
370       P=P+Q
380     wend
390   next
400   Q=Q+2*R
410 wend
420 '
430 '  sort file1
440 file1(C+1)=pack(0,0) ' dummy data
450 print:print "Now sorting"
460 print=print+"accept.txt"
470 print "( # of elements=";C;")"
480 print=print
490 open "sort.ubd" for create as file2(100000) word 8
500 gosub *Sort(1,C,0)
510 print:print "# of not sorted intervals=";C1
520 C2=C1
530 for I=1 to C2
540   gosub *Sort(member(file2(I),1),member(file2(I),2),0)
550 next
560 if C1>C2 then
570   :print=print+"accept.txt"
580   :print "Not sorted yet"
590   :print=print
600 endif
610 '
620 '  list possibly acceptable values
630 print
640 print "Now checking"
650 print=print+"accept.txt"
660 D=1
670 while D<=C
680   P=member(file1(D),1)
690   V=0
```

```
 700     if P@R=1 then V=log(R)
 710     while member(file1(D),1)=P
 720       Q=member(file1(D),2)
 730       V=V+log(Q)
 740       N%=2
 750       while modpow(P,R,Q^N%)=1
 760         V=V+log(Q)
 770         N%=N%+1
 780       wend
 790       D=D+1
 800     wend
 810     if V>(R-1)*log(P) then print P
 820   wend
 830   '
 840   '  closing
 850   print "Checked R=";R;"in";time
 860   print
 870   print=print
 880   close:kill "pairs.ubd":kill "sort.ubd"
 890   R=nxtprm(R)
 900   if R<=T then goto 170
 910   end
 920   '
 930   '  quick sort
 940   '
 950   '    Remark. N represents the number of recursion levels.
 960   '    There is a limit of N on UBASIC,
 970   '    so we need some steps to sort data.
 980   '
 990   *Sort(A,B,N)
1000   local I,J,M,TMP
1010   if A>=B then goto 1320
1020   if N>48 then
1030     :C1=C1+1
1040     :file2(C1)=pack(A,B)
1050     :goto 1320
1060   endif
1070   print "                         ";chr(13);
1080   print A;
1090   I=A
1100   '
1110   '  determine a key element
1120   M=member(file1(I),1)
1130   while and{I<=B,member(file1(I),1)=M}:I=I+1:wend
1140   if I>B then goto 1320
1150   if member(file1(I),1)>M then M=member(file1(I),1)
```

```
1160  '
1170  I=A:J=B
1180  while I<J
1190    while member(file1(I),1)<M:I=I+1:wend
1200    while member(file1(J),1)>=M:J=J-1:wend
1210    '
1220    '  switch the elements
1230    if I<J then
1240      :TMP=file1(I)
1250      :file1(I)=file1(J)
1260      :file1(J)=TMP
1270    endif
1280    '
1290  wend
1300  gosub *Sort(A,I-1,N+1)
1310  gosub *Sort(J+1,B,N+1)
1320  return
1330  '
1340  '  generator of (Z/QZ)^*
1350  '
1360  fnGEN(Q)
1370  local A,QQ,P
1380  A=2
1390  QQ=Q-1
1400  while QQ>1
1410    P=prmdiv(QQ)
1420    if modpow(A,(Q-1)//P,Q)=1 then A=A+1:goto 1390
1430    while QQ@P=0:QQ=QQ//P:wend
1440  wend
1450  return(A)
```

A.6.10 accept2.ub

```
  10  '
  20  '  ACCEPT2.UB
  30  '
  40  '  originally produced by Paul M. Jenkins as "PROP9.UB"
  50  '  modified by Takeshi GOTO (Oct. 2005)
  60  '    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
  70  '
  80  '  used to find acceptable values \Phi_R(P) for P<100
  90  '  output file: ACCEPT2.TXT
 100  '  expected CPU time: about ten minutes for one value of P
 110  '
 120  V=0
 130  input "Enter P";Start
```

```
140 P=nxtprm(Start-1)
150 input "Enter Rp";Rp
160 clr time
170 print=print+"accept2.txt"
180 R=7
190 while R<Rp
200   V=0
210   if P@R=1 then V=V+log(R)
220   X=2*R+1
230   while X<100000000
240     if prmdiv(X)<X then goto 310
250     if modpow(P,R,X)=1 then V=V+log(X)
260       :if modpow(P,R,X^2)=1 then V=V+log(X)
270         :if modpow(P,R,X^3)=1 then V=V+log(X)
280           :print X;"^3 divides";P;"^";R;"-1"
290         :endif
300       :endif
310     endif
320     X=X+2*R
330   wend
340   if V>=(R-1)*log(P) then
350     :print "Look at P=";P;"and R=";R
360   endif
370   R=nxtprm(R)
380 wend
390 print "Checked P=";P;"for Rp <";Rp;"in";time
400 print=print
410 end
```

A.6.11 ptester.ub

```
10  '
20  '  PTESTER.UB
30  '
40  '  originally produced by Paul M. Jenkins
50  '  modified by Takeshi GOTO (Oct. 2005)
60  '    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
70  '
80  '  used to calculate P^*
90  '  expected CPU time: about thirty minutes
100 '
110 point -8
120 Pcount=0
130 Pproduct=1
140 P=41
150 while P<10^8
```

```
160     print P;
170     print "                       ";chr(13);
180     Pcount=Pcount+1
190     Pproduct=Pproduct*P/(P-1)
200     P=nxtprm(P)
210   wend
220   print "Pcount=";Pcount
230   print "Pproduct=";Pproduct
240   end
```

A.6.12 stester.ub

```
 10   '
 20   '  STESTER.UB
 30   '
 40   '  originally produced by Paul M. Jenkins
 50   '  modified by Takeshi GOTO (Oct. 2005)
 60   '    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
 70   '
 80   '  used to calculate S^*
 90   '  expected CPU time: about ten minutes
100   '
110   point -8
120   Scount=0
130   Sproduct=1
140   N=41
150   while N<10^8
160     if and{N@3>1 and N@5>1} then
170       :print N;
180       :print "                      ";chr(13);
190       :Scount=Scount+1
200       :Sproduct=Sproduct*N/(N-1)
210     endif
220     N=nxtprm(N)
230   wend
240   print "Scount=";Scount
250   print "Sproduct=";Sproduct
260   end
```

A.6.13 ttester.ub

```
 10   '
 20   '  TTESTER.UB
 30   '
 40   '  originally produced by Paul M. Jenkins
 50   '  modified by Takeshi GOTO (Oct. 2005)
```

```
 60 '    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
 70 '
 80 '  used to calculate T^*
 90 '  expected CPU time: about five minutes
100 '
110 point -8
120 Tcount=0
130 Tproduct=1
140 N=41
150 while N<10^8
160   if and{N@3=1,N@5=1} then
170     :print N;
180     :print "                       ";chr(13);
190     :Tcount=Tcount+1
200     :Tproduct=Tproduct*N/(N-1)
210   endif
220   N=nxtprm(N)
230 wend
240 print "Tcount=";Tcount
250 print "Tproduct=";Tproduct
260 end
```

A.6.14 utester.ub

```
 10 '
 20 '  UTESTER.UB
 30 '
 40 '  by Takeshi GOTO (Oct. 2005)
 50 '    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
 60 '
 70 '  used to calculate U^*
 80 '  expected CPU time: about two hours
 90 '
100 point -8
110 clr time
120 Ucount=0
130 Uproduct=1
140 '
150 '  write all pairs (P,Q) satisfying
160 '  P \equiv 1 (mod 3), P \not \equiv 1 (mod 5),
170 '  Q | \Phi_5(P) and P,Q<10^8 on file1
180 open "pairs.ubd" for create as file1(2^32-1) word 8
190 Q=11
200 while Q<100000000
210   if prmdiv(Q)<Q then goto 340
220   print "                         ";chr(13);
```

```
230     print " Q=";Q;
240     G=fnGEN(Q)
250     W=modpow(G,(Q-1)//5,Q):P=1
260     for I%=1 to 4
270       P=(P*W)@Q
280       while P<100000000
290         if or{prmdiv(P)<P,P<40,P@3<>1,P@5=1} then goto 310
300         C=C+1:file1(C)=pack(P,Q)
310         P=P+Q
320       wend
330     next
340     Q=Q+10
350   wend
360   '
370   '  sort file1
380   file1(C+1)=pack(0,0) ' dummy data
390   print:print "Now sorting ( # of elements=";C;")            "
400   open "sort.ubd" for create as file2(100000) word 8
410   gosub *Sort(1,C,0)
420   print:print "# of not sorted intervals=";C1
430   C2=C1
440   for I=1 to C2
450     gosub *Sort(member(file2(I),1),member(file2(I),2),0)
460   next
470   if C1>C2 then print "Not sorted yet"
480   '
490   print
500   D=1
510   P=43
520   while P<100000000
530     if or{prmdiv(P)<P,P@5=1} then goto 700
540     V=1
550     if member(file1(D),1)>P then goto 660
560     while member(file1(D),1)=P
570       Q=member(file1(D),2)
580       V=V*Q
590       N%=2
600       while modpow(P,5,Q^N%)=1
610         V=V*Q
620         N%=N%+1
630       wend
640       D=D+1
650     wend
660     if V<P^4+P^3+P^2+P+1 then
670       :Ucount=Ucount+1
680       :Uproduct=Uproduct*P/(P-1)
```

```
690     endif
700     P=P+6
710   wend
720   '
730   '  closing
740   print "Ucount=";Ucount
750   print "Uproduct=";Uproduct
760   print "Calculated in";time
770   close:kill "pairs.ubd":kill "sort.ubd"
780   end
790   '
800   '  quick sort
810   '
820   '    Remark. N represents the number of recursion levels.
830   '    There is a limit of N on UBASIC,
840   '    so we need some steps to sort data.
850   '
860   *Sort(A,B,N)
870   local I,J,M,TMP
880   if A>=B then goto 1190
890   if N>48 then
900     :C1=C1+1
910     :file2(C1)=pack(A,B)
920     :goto 1190
930   endif
940   print "                         ";chr(13);
950   print A;
960   I=A
970   '
980   '  determine a key element
990   M=member(file1(I),1)
1000  while and{I<=B,member(file1(I),1)=M}:I=I+1:wend
1010  if I>B then goto 1190
1020  if member(file1(I),1)>M then M=member(file1(I),1)
1030  '
1040  I=A:J=B
1050  while I<J
1060    while member(file1(I),1)<M:I=I+1:wend
1070    while member(file1(J),1)>=M:J=J-1:wend
1080    '
1090    '  switch the elements
1100    if I<J then
1110      :TMP=file1(I)
1120      :file1(I)=file1(J)
1130      :file1(J)=TMP
1140    endif
```

```
1150    '
1160  wend
1170  gosub *Sort(A,I-1,N+1)
1180  gosub *Sort(J+1,B,N+1)
1190  return
1200  '
1210  '  generator of (Z/QZ)^*
1220  '
1230  fnGEN(Q)
1240  local A,QQ,P
1250  A=2
1260  QQ=Q-1
1270  while QQ>1
1280    P=prmdiv(QQ)
1290    if modpow(A,(Q-1)//P,Q)=1 then A=A+1:goto 1260
1300    while QQ@P=0:QQ=QQ//P:wend
1310  wend
1320  return(A)
```

A.6.15 vtester.ub

```
  10  '
  20  '  VTESTER.UB
  30  '
  40  '  by Takeshi GOTO (Oct. 2005)
  50  '    URL: http://www.ma.noda.tus.ac.jp/u/tg/perfect.html
  60  '
  70  '  used to calculate V^*
  80  '  expected CPU time: about an hour
  90  '
 100  point -8
 110  clr time
 120  Vcount=0
 130  Vproduct=1
 140  '
 150  '  write all pairs (P,Q) satisfying
 160  '  P \not \equiv 1 (mod 3), P \equiv 1 (mod 5),
 170  '  Q | \Phi_3(P) and P,Q<10^8 on file1
 180  open "pairs.ubd" for create as file1(2^32-1) word 8
 190  Q=7
 200  while Q<100000000
 210    if prmdiv(Q)<Q then goto 340
 220    print "                          ";chr(13);
 230    print " Q=";Q;
 240    G=fnGEN(Q)
 250    W=modpow(G,(Q-1)//3,Q):P=1
```

```
260     for I%=1 to 2
270       P=(P*W)@Q
280       while P<100000000
290         if or{prmdiv(P)<P,P<40,P@3=1,P@5<>1} then goto 310
300         C=C+1:file1(C)=pack(P,Q)
310         P=P+Q
320       wend
330     next
340     Q=Q+6
350   wend
360   '
370   '  sort file1
380   file1(C+1)=pack(0,0) ' dummy data
390   print:print "Now sorting ( # of elements=";C;")             "
400   open "sort.ubd" for create as file2(100000) word 8
410   gosub *Sort(1,C,0)
420   print:print "# of not sorted intervals=";C1
430   C2=C1
440   for I=1 to C2
450     gosub *Sort(member(file2(I),1),member(file2(I),2),0)
460   next
470   if C1>C2 then print "Not sorted yet"
480   '
490   print
500   D=1
510   P=41
520   while P<100000000
530     if or{prmdiv(P)<P,P@3=1} then goto 700
540     V=1
550     if member(file1(D),1)>P then goto 660
560     while member(file1(D),1)=P
570       Q=member(file1(D),2)
580       V=V*Q
590       N%=2
600       while modpow(P,3,Q^N%)=1
610         V=V*Q
620         N%=N%+1
630       wend
640       D=D+1
650     wend
660     if V<P^2+P+1 then
670       :Vcount=Vcount+1
680       :Vproduct=Vproduct*P/(P-1)
690     endif
700     P=P+10
710   wend
```

```
720  ’
730  ’  closing
740  print "Vcount=";Vcount
750  print "Vproduct=";Vproduct
760  print "Calculated in";time
770  close:kill "pairs.ubd":kill "sort.ubd"
780  end
790  ’
800  ’  quick sort
810  ’
820  ’    Remark. N represents the number of recursion levels.
830  ’    There is a limit of N on UBASIC,
840  ’    so we need some steps to sort data.
850  ’
860  *Sort(A,B,N)
870  local I,J,M,TMP
880  if A>=B then goto 1190
890  if N>48 then
900    :C1=C1+1
910    :file2(C1)=pack(A,B)
920    :goto 1190
930  endif
940  print "                         ";chr(13);
950  print A;
960  I=A
970  ’
980  ’  determine a key element
990  M=member(file1(I),1)
1000 while and{I<=B,member(file1(I),1)=M}:I=I+1:wend
1010 if I>B then goto 1190
1020 if member(file1(I),1)>M then M=member(file1(I),1)
1030 ’
1040 I=A:J=B
1050 while I<J
1060   while member(file1(I),1)<M:I=I+1:wend
1070   while member(file1(J),1)>=M:J=J-1:wend
1080   ’
1090   ’  switch the elements
1100   if I<J then
1110     :TMP=file1(I)
1120     :file1(I)=file1(J)
1130     :file1(J)=TMP
1140   endif
1150   ’
1160 wend
1170 gosub *Sort(A,I-1,N+1)
```

```
1180   gosub *Sort(J+1,B,N+1)
1190   return
1200   '
1210   '  generator of (Z/QZ)^*
1220   '
1230   fnGEN(Q)
1240   local A,QQ,P
1250   A=2
1260   QQ=Q-1
1270   while QQ>1
1280     P=prmdiv(QQ)
1290     if modpow(A,(Q-1)//P,Q)=1 then A=A+1:goto 1260
1300     while QQ@P=0:QQ=QQ//P:wend
1310   wend
1320   return(A)
```

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